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Obtaining a Generalized Index of Bank Competitiveness Using a Fuzzy Approach

Abstract: The article is devoted to developing a definition of the indicator of the bank's competitiveness which based on the theory of fuzzy sets and neural networks techniques. Uncertainties that have a place when considering and analyzing the components of evaluating the success and effectiveness of the bank have been considered and analyzed. The sequence of construction and structure for generalizing parameter of bank competitiveness are presented and grounded. Stages of obtaining an integrated assessment of bank competitiveness by sequential application of fuzzy logic and neural networks approaches are determined and described. Corresponding fuzzy terms, membership functions and fuzzy inference rules are described. Overall sequence and steps to resolve the problem are processed. The practical implementation of the summary fuzzy inference of the bank's competitiveness is given. In particular, numerical calculations on the proposed model for Ukrainian commercial bank "Khreshchatyk" was carried out. Comparison of obtained evaluation results for the competitiveness of specified bank with available data and other scientific information sources showed their compliance with factual situation. In this way, the expediency of application fuzzy modeling has been confirmed to determine the generalized indicators of bank competitiveness. Adequacy and accuracy of the proposed model and the results of calculations were proved. The proposed approach is quite general. This or similar model can be successfully used in other tasks of building and generalized evaluation of integrated indicators for the presence of several local, individual parameters for different economic processes and tasks.

Keywords: bank competitiveness, fuzzy modeling, bank indicators, uncertainty in economics, bank service estimations, neural networks modeling, constructing general indicators

JEL classification: C45; C69; G21

1. Introduction

One of the conditions for stabilization of the economy is a well-developed banking sector (Asanovic, 2017; Luburic & Fabris, 2017; Ponomarenko, Gontareva & Dorokhov, 2014). In turn, the financial health of banks depends on their competitiveness, which requires correct determination and management (Degl'Innocenti, Kourtzidis, Sevic & Tzeremes, 2017; Goncharuk, 2016; Lapteacru, 2014; Menicucci, Paolucci, Zain & Rasit, 2016).

At present, a bank is a complicated socio-economic system. Such systems are best described by characteristics that have a variety of uncertainties. Nowadays, the theory of fuzzy sets is one of the most effective intelligent technologies for modeling and design of complex systems under uncertainty conditions. Therefore, the theory of fuzzy sets has been successfully applied in a number of recent studies on the banking and financial sector (Barros, 2016; Braendle & Sepasi, 2014; Kukal & Vanquang, 2014; Mandic, Delibasic, Knezevic & Benkovic, 2014).

The uncertainty of input data may be caused by results of research, forecasted data, the incompleteness of information, rounding errors. It should be noted that a large amount of data on the competitiveness of banks are usually obtained from experts (expert evaluations). Such data are often subjective, and therefore difficult to analyze and formalize. (Fabris & Vujanovic, 2017; Malyaretz, Dorokhov & Dorokhova, 2018; Omelchenko, Dorokhov, Kolodiziev & Dorokhova, 2018).

Determining bank competitiveness is necessary to develop a generalizing index, which would take into account the complex interactions between components of competitiveness and influence of external factors. Therefore, the specification of a model of generalizing index involves the use of fuzzy sets tools and neural networks that underlie many expert systems (Fernando, Ferreira, Jalali, Ferreira, Stankevičienė & Marques, 2016; Hooman, Marthandan, Yusoff & Omid, 2016; Rezaei & Ketabi, (2016).

2. Formulation of the general problem

Many famous mathematicians and economists recommend using fuzzy sets tools in order to develop generalizing indexes in the economy and argued about it in their works (Anfilatov, Emelyanov & Kukushkin, 2003; Borisov, Kruglov & Fedulov, 2007; Yarushkina & Afanasyeva, 2007). Using fuzzy sets implies mathematical formalization of fuzzy estimates in the form of linguistic variables for building models for processing these estimates as membership functions compositions that have simple linguistic interpretation.

As a result, end users obtain the opportunity to work with domain-specific linguistic terms, which are represented as numbers at the level of computer calculations. Such an approach makes approximates but simple qualitative ways for describing complex systems (Dorokhov, Chernov, Dorokhova & Streimkis, 2018; Dorokhov & Dorokhova, 2011).

The main idea and purpose of using fuzzy sets in economic research is the formalization by means of fuzzy modeling of estimates of economic processes, their components, and elements, properties, characteristics and behavior of economic systems, objects and subjects in conditions of uncertainty, insufficiency, low reliability of the initial information.

The problem of constructing general indicators has always been actual in measurement for objects of different nature. For example, the level of values of competitiveness can generate conclusion about the financial condition of a bank, compare the given indicator in the previous period, compare a similar indicator with another bank. Aggregated description contains less information than the original (initial) data. Herewith, at the same time, useful information is saved, but excessive information is reduced. (Ponomarenko & Malyarets, 2009).

Most often a general indicator of bank competitiveness is based on convolutions of indicators in an additive or multiplicative form. The additive convolution executed by the formula:

$$I_{\Sigma} = \sum_{i=1}^{n} \lambda_i x_i, \quad (1)$$

where x_i – value of the i-th indicator, which is measured on the scale of intervals or relations; λ_i – the coefficient of indicator significance.

Multiplicative convolution individual indicators executed by formula:

$$I_{\Pi} = \prod_{i=1}^{n} {x_i}^{\lambda_i}, \quad (2)$$

Most often used additive convolution in modified form:

$$I_{\Sigma} = \left(\sum_{i=1}^{n} \lambda_i |x_i - \alpha_i|^k\right)^{\frac{1}{k}}, \quad (3)$$

where α_i – etalon value of indicator; k – parameter.

Specialists of mathematical methods in economics propose the convolution of signs to be used as a summary measure of quality (Ponomarenko & Malyarets, 2009). The logic of calculating this indicator in addition to typical problems of constructing general indicators, includes the following stages: generalization of theoretical and practical knowledge according to this sign of system, realization of descriptive statistics of characteristics; formation scale for conversion of indicator values; determination of the main points for phase changes of indicator values; determining of separate functions for transformations of characteristics.

The mentioned experts propose to use functions of values transformations, which have the following form. For bilateral asymmetrical development trends of characteristics:

$$y_{ij} = \begin{cases} 100 \cdot e^{-3\left(\frac{x_{ij}-a_i}{b_i-a_i}\right)^2}, & for\ x_{ij} \le a_i, b_i < a_i, \\ 100 \cdot e^{-3\left(\frac{x_{ij}-a_i}{c_i-a_i}\right)^2}, & for\ x_{ij} \ge a_i, c_i > a_i, \end{cases}$$

where a_i, b_i, c_i – benchmark values: a_i – the best indicator value for x_{ij}, at which conversion function reaches the maximum value 1 (100%); b_i, c_i (b_i<c_i) – unsatisfactory indicator value x_{ij} (on opposite sides of the best), at which conversion function takes the value, not more than 0,05 (5%).

When symmetric tendencies of the development of characteristics conversion function take the value 1 (100%) at $a_i = \frac{b_i + c_i}{2}$. Then the form of a function simplified to: $y_{ij} = 100 \cdot e^{-3\left(\frac{x_{ij}-a_i}{b_i-a_i}\right)^2}$ or $y_{ij} = 100 \cdot e^{-3\left(\frac{x_{ij}-a_i}{c_i-a_i}\right)^2}$. For unilateral types the development of characteristics monotonic function can be built: $y_{ij} = \frac{100}{1+e^{-\frac{x_{ij}-p_i}{q_i-p_i}}}$, where q_1– the value of the indicator x_{ij}, at which conversion function takes the value not less than 0,95 (95%); p_i - the value of the indicator x_{ij}, t which conversion function takes the value 0,5 (50%).

The converted values of economic indicators are comparable with each other in both static and dynamic situations of measuring economic indicators, and they are objective measures of the characteristics of objects in the economy. However, to objectively determine the level of bank competitiveness, it is necessary to take into account the uncertainty of all components and its form, and this will require more than just the aforementioned methods.

3. The general approach to solving the problem

Based on the above, we can formulate the purpose and objectives of our study. They consist of the development of a methodology for constructing a summary index of competitiveness of a bank, taking into account the complex interrelationships between internal components and the difficultly formalized influence of external factors. The use of approaches based on the theory of fuzzy sets and the use of appropriate computer tools of fuzzy modeling is proposed as a tool for modeling, carrying out calculations and subsequent ones.

As is well known, there are five basic methods for implementation of a synthesis of fuzzy inference in fuzzy sets theory (Yarushkina, 2004).

Method 1: Mamdani algorithm. In this algorithm, the implication is modeled by a minimum, and aggregation is modeled by a maximum. Also, this uses the minimax composition of fuzzy sets. Each subsequent step of algorithm gets the input value of the previous step. The input of the algorithm receives quantitative values, the output also produces quantitative values. However, at the intermediate stage of fuzzification, the values are converted to fuzzy with the definition of the degree of their truth, that is, the parameters of the membership functions are defined for the left parts of each rule (prerequisites). Fuzzy output formed in such a way: at first are determined levels of "cut-off" for the left side of each rule, then is found "truncated" membership functions. The next stage of algorithm Mamdani is the composition of obtained truncated functions. And the last stage is a process of defuzzification – bringing data to clarity, for example, by the method of the middle center.

Method 2: Tsukamoto algorithm. Initial presuppositions the same as in the previous algorithm, but it is believed that the membership functions are monotonous.

Method 3: Sugeno algorithm. It is believed that the right parts of inference rules are linear functions.

Method 4: Larsen algorithm. The fuzzy implication is modeled using a multiplication operation.

Method 5: Simplified fuzzy logic algorithm. Initial rules are specified as: if $X \in A_i$ i $Y \in B_i$, then $z = Z_i$, where Z_i – crisp number.

Experts in the field of fuzzy methods and their application for the analysis of economic systems in their works (Matviychuk, 2005) also analyzed competitiveness (using fuzzy logic and neural networks), but only for enterprises. They recommend to carry out the determination of generalizing indicator by 5 steps.

At the same time, at the 1st stage, certain indicators are determined, which are the main signs of the competitiveness of an enterprise (in particular, a bank). At the 2nd stage are formed linguistic variables (input and output) and given the unified scale of their assessment in the form of qualitative terms. The third stage is to build membership functions. At 4th stage is formed a set of rules, by means of which, through the mechanism of fuzzy-logical conclusions, could be determined the level of competitiveness of the enterprise.

The final, 5th stage, is to evaluate the current level of indicators (input and output) by financial reporting for various time periods. at the same time, the values of the controlled parameters, which exactly fall within the intervals defined for certain terms, will unambiguously correspond to these (one or another) terms. But if the value of criterion is located in the interval between two terms, then it will match to the corresponding term, whose membership function for a given level of the indicator is the largest.

To develop a generalized indicator of bank competitiveness, we will use the recommendations (Braendle & Sepasi, 2014) for simplification of fuzzy inference algorithm (method 5) with a practical use for the calculation special program tools for fuzzy computing in *MS Excel – Fexcel* (Cveshnikov & Bocharnikov, 2007). Mentioned program *Fexcel* provides all the necessary components of processing of fuzzy numbers: mathematical, software, information, linguistic.

Therefore, when developing a generalized indicator of bank competitiveness, we recommend sticking to the following logical sequence of actions. For each component of bank competitiveness: competitiveness of banking services (products); management of bank; resources of the bank; ability to wage competitive fight; factors of external environment, is formed linguistic variable (Koybichuk, 2012). Is determined intersection of current value using maximin composition.

The maximum level of intersection defines the quantity of current value. The next step is the formation of a measure for indicators importance using membership functions. Next again should be applied maximin composition of fuzzy sets. A finally we have obtained a level of current value for generalizing index of competitiveness for considered a bank.

4. Description and formalization of components of bank competitiveness: choice of selected indicators and formation data array

Let us consider a proposed logic approach for determining a summary measure of competitiveness for bank "Khreshchatyk" as of the 2015 year. Indicators $X_{ij},\ i=\overline{1,N},\ j=\overline{1,M_i}$, singled out on the basis of theoretical and logical analysis to N groups. So, all individual indicators that describe the bank competitiveness are structured to groups (Malyaretz et al., 2018). Specifically, competitiveness of banking products described by indicators:

$$X_1 = f_1(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \quad (4)$$

where x_1 – consumer property of service, x_2 – the cost of providing service, x_3 – terms of providing service, x_4 – the speed of providing service, x_5 – ways to promote services, x_6 – the breadth of assortment, x_7 – branching of marketing network, x_8 – the quality of after sales service.

Management of bank described by indicators: x_9 – innovative banking services, x_{10} – experience in project management, x_{11} – the level of management, x_{12} – period of the bank working on the market, x_{13} – number of branches, x_{14} – number of offices, x_{15} – effective use of technologies and x_{16} – developed system of collaboration:

$$X_2 = f_2(x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}) \quad (5)$$

Bank resources defined by parameters: x_{17} – own funds, x_{18} – borrowed funds, x_{19} – borrowed bank financial resources:

$$X_3 = f_3(x_{17}, x_{18}, x_{19}) \quad (6)$$

Ability to wage competitive struggle described by parameters: x_{20} – profitability of assets, x_{21} – profitability of capital, x_{22} – total profitability of assets, x_{23} – net interest margin, x_{24} – net spread, x_{25} – aggregate share of net interest income and net commission income in operating profit, x_{26} – borrowed funds on terms of subordinated debt, x_{27} – the interest rate on subordinated debt in national currency, x_{28} – the interest rate on subordinated debt in US dollars, x_{29} – the interest rate on subordinated debt in euro, x_{30} – GAP, x_{31} – the share of equity in net assetsx, x_{32} – ratio coefficient of credit portfolio to bank liabilities, x_{33} – the share of fixed assets and intangible assets in net assets, x_{34} – the share of individuals' deposits in liabilities, x_{35} – the share of reserves for credit transactions in the loan portfolio, x_{36} – adequacy ratio (adequacy) of regulatory capital, x_{37} – interrelation coefficient of regulatory capital to total assets of capital, x_{38} – the maximum size of credit risk to one counteragent (established in order to limit credit risk, arising as a result of failure to fulfill individual counteragents their obligations), x_{39} – the coefficient of financial leverage, x_{40} – the presence of foreign capital:

$$X_4 = f_4(x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{40}) \quad (7)$$

The indicator of the external environment X_5 includes the following: x_{41} – the monetary base, x_{42} – the producer price index for industrial production, x_{43} – the rate of change of cash course, x_{44} – the rate of refinancing, x_{45} – the average rate of bank deposit resources, x_{46} – unemployment rate (by ILO methodology), x_{47} – presence of foreign capital in the banking system:

$$X_5 = f_5(x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, x_{46}, x_{47}) \quad (8)$$

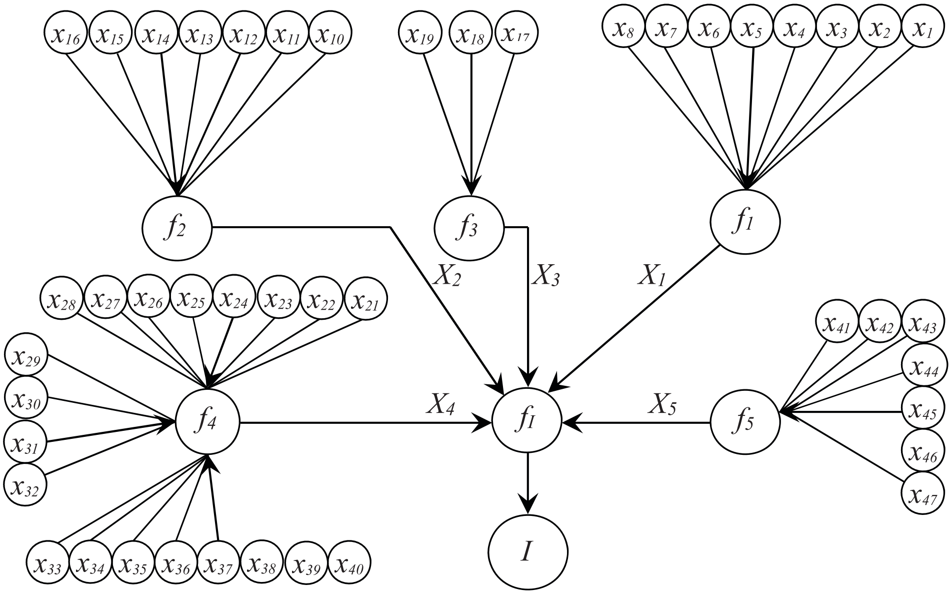
On the basis of the defined components can be calculated the level of bank competitiveness in general:

$$I = f_I(X_1, X_2, X_3, X_4, X_5) \quad (9)$$

A set of specific indicators to describe the components of bank competitiveness, as a list of components may be different for various banks. When constructing a fuzzy model we use the method neural networks because it is proven that these networks are universal approximators and carry out the withdrawal based on fuzzy logic apparatus (Yarushkina, Afanasyeva & Perfyleva, 2010)

The fuzzy neural network typically consists of four layers: a layer for fuzzification of input variables, layer for aggregation of values of activation conditions, layer for aggregation of fuzzy rules, and an output layer (Leonenkov, 2005). A structural and functional descriptive model of bank competitiveness corresponding to the ratio (1–6) and elements of conceptual scheme of bank competitiveness (Koybichuk, 2012) can be represented as a tree of logical conclusion for generalizing index of bank competitiveness (Figure1).

Figure 1. Tree of logical conclusion for generalizing index of bank competitiveness



5. Determining linguistic variables and construction of membership functions

The linguistic variable takes values from the set of words or combinations of words of some natural language and described by the following "fives": $< x, T, X, G, M >$ where x – the name of the variable, T – term set, each element of which is given by fuzzy set on a universal set X; G – syntax rules (often in form of grammar), which generate names of terms; M – semantic rules that define membership functions for fuzzy terms, generated by syntactical rules from G.

For the purpose of evaluating and processing values of indicators $X_{ij,}\ i = \overline{1, N}, j = \overline{1, M_i}$, will form a single scale with five qualitative terms: VL – very low level of the indicator X_{ij}, L – low level of the indicator X_{ij}, M – middle level of the indicator X_{ij}, H – high level of the indicator X_{ij}, VH – very high level of the indicator X_{ij}.

To evaluate the value of output linguistic variable I, which represents a generalizing index of bank competitiveness, it is proposed to use those same terms. At this stage, the form for membership functions of fuzzy terms for controlled parameters $X_{ij,}\ i = \overline{1, N}, j = \overline{1, M_i}$, and output variable I must also be specified.

In the classical theory of fuzzy sets, three algorithms are most often used to construct membership functions (Shtovba, 2006). The first algorithm uses expert information while using methods of statistical processing of expert information (Borisov & Krumberg, 1990) to summarize the collective opinions of experts regarding the distribution of elements on the sets and methods of paired comparisons (Rothstein, 1999) to construct membership function by a survey of a single expert.

The second algorithm is based on the parametric identification of fuzzy models by experimental data "inputs – output". When identification is optimized parameters of membership function in order to minimize differences between experimental data and results of fuzzy modeling.

The basis of the third algorithm for constructing membership functions is the use of the results of observations distribution. This task is similar to the construction of distribution function for a random variable by experimental data. In the statistics, the histogram method is used for this purpose. By this method, the membership functions for the corresponding subnormal (for which $\mu(x)<1$) fuzzy set can be constructed.

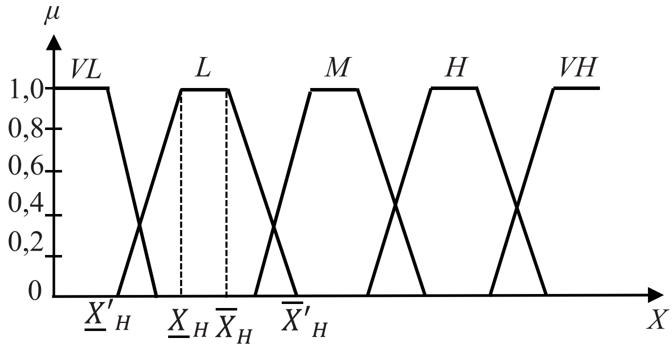
In this study, we will apply the second algorithm to construct membership functions. First, it is necessary to determine the possible range of monitored (controlled) parameters x_{ij} and output variable *I*. We will apply trapezoidal membership functions, reflecting elements of set *X* (universal) to the plural of numbers in the range [0, 1], which indicate the degree of belonging of each element in different qualitative terms and in the program *FExcel* is constructed by using the tool *FuzzyFigure*.

The using of trapezoidal membership functions is due to the fact that it is with their application that you can set exact limits, in which the value of each parameter will definitely conform their own terms. Beyond these boundaries, controversial moments will be decided after learning model by values of membership functions of each term for all parameters.

The possibility and sufficiency of using trapezoid dependences for defining a fuzzy number based on a parametric approach have been justified in (Cveshnikov & Bocharnikov, 2007). Such trapezoid dependences provide a representation of a fuzzy number in the form of one of seven geometric figures: trapezoid, left-hand trapezoid, right-hand trapezoid, rectangle, isosceles triangle, left-hand triangle, right-hand triangle. To simplify the presentation of the methodology, trapezoidal membership functions also are considered in our research and modeling.

For indicators of investigated bank piecewise linear trapezoidal membership function defined on Universum *X*, as that was selected a closed interval of real numbers. The ranges of changing parameters $X_{ij,}\ i = \overline{1, N},\ j = \overline{1, M_i}$ on a single universal set *X* for building membership function consist of five fuzzy terms for input variable {very low-*VL(ДН)*, low-*L(H)*, middle-*M(C)*, high-*H(B)*, very high-*VH(ДВ)*} are shown in Figure 2.

Figure. 2. Fuzzy variable x_{ij} with the trapezoidal membership function



Analytical appearance for trapezoidal membership functions of fuzzy terms for input variable (Figure 2) is the next:

$$\mu^{ДН}(X) = \begin{cases} 1, & X \le \overline{X}_{ДН} \\ \dfrac{\overline{X}'_{ДН} - X}{\overline{X}'_{ДН} - \overline{X}_{ДН}}, & \overline{X}_{ДН} < X \le \overline{X}'_{ДН} \\ 0, & X > \overline{X}'_{ДН} \end{cases} \tag{10}$$

$$\mu^{Н}(X) = \begin{cases} 0, & X < \underline{X}'_{Н} \\ \dfrac{X - \underline{X}'_{Н}}{\underline{X}_{Н} - \underline{X}'_{Н}}, & \underline{X}'_{Н} \le X < \underline{X}_{Н} \\ 1, & \underline{X}_{Н} \le X \le \overline{X}_{Н} \\ \dfrac{\overline{X}'_{Н} - X}{\overline{X}'_{Н} - \overline{X}_{Н}}, & \overline{X}_{Н} \le X \le \overline{X}'_{Н} \\ 0, & X > \overline{X}'_{Н} \end{cases} \tag{11}$$

$$\mu^{C}(X)=\begin{cases}0, & X<\underline{X}'_C\\ \dfrac{X-\underline{X}'_C}{\underline{X}_C-\underline{X}'_C}, & \underline{X}'_C\le X<\underline{X}_C\\ 1, & \underline{X}_C\le X\le\overline{X}_C\\ \dfrac{\overline{X}'_C-X}{\overline{X}'_C-\overline{X}_C}, & \overline{X}_C\le X\le\overline{X}'_C\\ 0, & X>\overline{X}'_C\end{cases}\tag{12}$$

$$\mu^{B}(X)=\begin{cases}0, & X<\underline{X}_B\\ \dfrac{X-\underline{X}'_B}{\underline{X}_B-\underline{X}'_B}, & \underline{X}'_B\le X<\underline{X}_B\\ 1, & \underline{X}_B\le X\le\overline{X}_B\\ \dfrac{\overline{X}'_B-X}{\overline{X}'_B-\overline{X}_B}, & \overline{X}_B\le X\le\overline{X}'_B\\ 0, & X>\overline{X}'_B\end{cases}\tag{13}$$

$$\mu^{ДB}(X)=\begin{cases}0, & X<\underline{X}'_{ДB}\\ \dfrac{X-\underline{X}'_{ДB}}{\underline{X}_{ДB}-\underline{X}'_{ДB}}, & \underline{X}'_{ДB}\le X\le\underline{X}_{ДB}\\ 1, & X>\overline{X}_{ДB}\end{cases}\tag{14}$$

For each term *{VL, L, M, H, VH}* parameters $\underline{X}'$ and $\overline{X}'$ characterize the lower trapezoid basis, and parameters $\underline{X}$ and $\overline{X}$ – upper trapezoid basis. Furthermore, this membership function generates a normal convex fuzzy set with the carrier (interval) $(\underline{X}', \overline{X}')$, boundaries $(\underline{X}', \underline{X}) \cup (\overline{X}, \overline{X}')$ and core $(\underline{X}, \overline{X})$ as shown in (Leonenkov, 2005). Similarly are constructed functions of fuzzy terms *{VL, L, M, H, VH}* for output variable *I*.

6. Forming the ruleset

An expert system based on fuzzy knowledge must include a mechanism vague logical conclusion with the help of which could be determined bank competitiveness on the basis of all relevant information. A fragment of the set of decision rules used in the proposed model is given in Table 1.

Table 1: The knowledge base for determining bank competitiveness

Generalized values for groups of indicators					Weight	Output variable
X_1	X_2	X_3	X_4	X_5	ω	I
VH	*H*	*VH*	*VH*	*M*	ω^{Y}_{11}	*VH*
VH	*M*	*H*	*VH*	*VH*	ω^{Y}_{12}	
H	*M*	*VH*	*VH*	*H*	ω^{Y}_{13}	
H	*H*	*VH*	*VH*	*VH*	ω^{Y}_{14}	
…	…	…	…	…	…	…
VL	*L*	*VL*	*C*	*L*	ω^{Y}_{51}	*VL*
VL	*C*	*L*	*L*	*VL*	ω^{Y}_{52}	
H	*C*	*VL*	*L*	*VL*	ω^{Y}_{53}	
VL	*L*	*L*	*VL*	*VL*	ω^{Y}_{54}	

The mathematical form of a notation decision rule for definition of level VH (bank competitiveness) through membership function has the form:

$$\begin{aligned}\mu^{ДВ}(X_1,X_2,X_3,X_4,X_5) = {} & \omega^{I}_{11}[\mu^{ДВ}(X_1)\cdot\mu^{В}(X_2)\cdot\mu^{ДВ}(X_3)\cdot\mu^{ДВ}(X_4)\cdot\mu^{С}(X_5)]\,\text{v} \\ & \text{v}\ \omega^{I}_{12}[\mu^{ДВ}(X_1)\cdot\mu^{С}(X_2)\cdot\mu^{В}(X_3)\cdot\mu^{ДВ}(X_4)\cdot\mu^{ДВ}(X_5)]\ \text{v}\ \omega^{I}_{13}[\mu^{В}(X_1)\cdot\mu^{С}(X_2)\cdot\mu^{ДВ}(X_3) \\ & \cdot\mu^{ДВ}(X_4)\cdot\mu^{В}(X_5)]\ \text{v}\ \omega^{I}_{14}[\mu^{В}(X_1)\cdot\mu^{В}(X_2)\cdot\mu^{ДВ}(X_3)\cdot\mu^{ДВ}(X_4)\cdot\mu^{ДВ}(X_5)].\end{aligned} \quad (15)$$

In turn, each of the criteria X_1, … , X_5, which are generalized values of these sets of indicators, should be present in the form of mathematical relationships from input factors. For example, a fragment of the knowledge base for determination X_1 (competitiveness of banking service) according to the function (4) is presented in Table 2.

Table 2: The knowledge base for determining the level of X_1 (competitiveness of banking service)

Generalized values for groups of indicators								Weight	Output variable
X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	ω	X_1
VH	*H*	*VH*	*H*	*VH*	*H*	*VH*	*H*	$\omega^{X_1}_{11}$	*VH*
VH	*M*	*H*	*H*	*VH*	*VH*	*H*	*VH*	$\omega^{X_1}_{12}$	
H	*M*	*VH*	*VH*	*VH*	*VH*	*H*	*VH*	$\omega^{X_1}_{13}$	
	…	…		…				…	…
VL	*L*	*VL*	*L*	*M*	*VL*	*L*	*VL*	$\omega^{X_1}_{51}$	*VL*
VL	*M*	*L*	*VL*	*L*	*L*	*VL*	*VL*	$\omega^{X_1}_{52}$	
L	*M*	*VL*	*H*	*L*	*VL*	*L*	*VL*	$\omega^{X_1}_{53}$	

The mathematical form of a notation decision rule for definition of level *VH* (competitiveness of banking service) through membership function has the form:

$$\mu^{ДВ}(x_1,\dots,x_8) = \omega_{11}^{X_1}[\mu^{ДВ}(x_1)\cdot\mu^{В}(x_2)\cdot\mu^{ДВ}(x_3)\cdot\mu^{В}(x_4)\cdot\mu^{ДВ}(x_5)\cdot\mu^{В}(x_6)\cdot\mu^{ДВ}(x_7) \cdot\mu^{В}(x_8)]\vee\omega_{12}^{X_1}[\mu^{ДВ}(x_1)\cdot\mu^{С}(x_2)\cdot\mu^{В}(x_3)\cdot\mu^{В}(x_4)\cdot\mu^{ДВ}(x_5)\cdot\mu^{ДВ}(x_6)\cdot\mu^{В}(x_7)\cdot \cdot\mu^{ДВ}(x_8)]\vee\omega_{13}^{X_1}[\mu^{В}(x_1)\cdot\mu^{С}(x_2)\cdot\mu^{ДВ}(x_3)\cdot\mu^{ДВ}(x_4)\cdot\mu^{ДВ}(x_5)\cdot\mu^{ДВ}(x_6)\cdot\mu^{В}(x_7)\cdot \cdot\mu^{ДВ}(x_8)]. \quad (16)$$

The formation of a complete set of decision rules and (based on them) the construction of a system of fuzzy logical equations is carried out in a similar way. Thus (as a result) a model is formed to describe the behavior of the system in a natural language in the form of approximate reasoning. The final solution to the model is one for which the membership function of output variable *I* will be the biggest for given parameters $X_{ij,}\ i = \overline{1,N},\ j = \overline{1,M_i}$.

7. Calculating assessments of indicators level

At this stage, carried evaluation of current level for indicators $X_{ij,}\ i = \overline{1,N},\ j = \overline{1,M_i}$ and *I* at financial reporting and expert judgments for various time periods.

The values of monitored parameters $X_{ij,}\ i = \overline{1,N},\ j = \overline{1,M_i}$ which just fall into defined to its intervals $[\underline{X}_{ij}, \overline{X}_{ij}]$, will definitely match their terms. But if the parameter value is between two terms, then it will correspond to the term for which the membership function for the given indicator level is the largest. Levels of all terms for each of indicators $X_{ij,}\ i = \overline{1,N},\ j = \overline{1,M_i}$ i for a particular bank are determined in accordance with normative values for classical criteria. If for a particular indicator norms do not exist, levels of terms separated on the basis of conducted researches concerning values of indicators for bank competitiveness (Shiryaev, 2007; Shtovba, 2006) by comparing values of indicators for different banks in different time periods.

The indication area for bank competitiveness by using tools of descriptive statistics, methods of canonical and factor analysis (in mathematical package *Statgraphics*) has been formed and clarified in (Koybichuk, 2013). Parameter x_{47} (presence of foreign capital in the banking system) on the recommendations of descriptive statistics was excluded from the system because its coefficient of variation is less than 5%.

Classification of chosen variables, which meet to services (products) competitiveness of bank "Khreshchatyk" (for the year of 2015) based on values that form indication area for complicated indicator «competitiveness of banking services», is shown in Table 3.

Table 3: Classification of indicators by levels

Parameter (indicator)	Indicator value that corresponds to the term				
	VL	*L*	*M*	*H*	*VH*
X_1	6-20	20-34	34-54,25	54,25-74,5	74,5-106,5
X_2	4-18,25	18,25-32,5	32,5-57,25	57,25-69,63	69,63-82
X_3	2-25	25-46	46-67	67-96	96-114,5
X_4	2-10	10-37	37-56,25	56,25-75,5	75,5-107
X_5	1,5-14,5	14,5-37,5	37,5-60,5	60,5-93	93-112
X_6	4-18,5	18,5-33	33-52,25	52,25-71,5	71,5-102,5
X_7	2,5-14,5	14,5-35	35-55,5	55,5-79,25	79,25-103,5
X_8	4,5-28	28-50,75	50,75-65,5	65,5-80,3	80,3-110

By using syntax and semantic rules (Tables 2, 3), that define membership functions of fuzzy terms (generated by syntactic rules) the level of competitiveness for banking products of bank "Khreshchatyk" for 2015 has been established (Table 4).

Table 4: Indicators X_1 Degree of importance for services competitiveness of bank "Khreshchatyk" for 2015

Indicator	Average value of the indicator for the 2015 year	Terms	indicator	Average value of the indicator for the 2015 year	Terms
X_1	63,25	*H*	X_5	57,13	*M*
X_2	37,75	*M*	X_6	79,25	*VH*
X_3	110,25	*VH*	X_7	57,25	*V*
X_4	73,75	*H*	X_8	73,5	*V*

Then for each indicator in Table 4 the intersection of current value using maximin composition is determined. Calculations were carried out in software product for MS Excel – Fexcel (Cveshnikov & Bocharnikov, 2007).

Mathematical notation of decisive rule to determine the level H of banking products competitiveness X_1 has appearance:

$$\mu^{\text{В}}(x_1,\ldots,x_8) = \omega_{11}^{X_1}[\mu^{\text{В}}(x_1)\cdot\mu^{\text{С}}(x_2)\cdot\mu^{\text{ДВ}}(x_3)\cdot\mu^{\text{В}}(x_4)\cdot\mu^{\text{С}}(x_5)\cdot\mu^{\text{ДВ}}(x_6)\cdot\mu^{\text{В}}(x_7)\cdot \cdot\mu^{\text{В}}(x_8)]. \quad (17)$$

Similarly to Tab. 4, a measure of importance was formed for indicators X_2, X_3 , $X_{4,}$ X_5 . For neural network (Yarushkina et al., 2010) is necessary that incoming data be belonged to the range [0…1], whereas data of studied area for bank competitiveness indicators can belong to any range [min … max], where min and max are the smallest and the largest values of the researched indicator.

So input range should be normalized. The easiest way to normalization are:

$$\tilde{x}_i = \frac{x_i - min}{max - min}, \tag{18}$$

where x_i – the output value of the i-th parameter, $\tilde{x}_i$ – value, which is fed to the input of the neural network.

This method of normalization has several disadvantages, for example, distribution of input parameters can be extremely uneven, which leads to deterioration of the quality of the training model.

Therefore a number of researchers suggest for time series modeling by neural networks and or forecasting of value level of time series on the basis of previous values (Barsky, 2004; Shiryaev, 2007; Yarushkina, 2004) to use normalization through the function of the form:

$$\tilde{x}_i = \frac{1}{1 - e^{-x}}, \tag{19}$$

8. Results and conclusions

So, as a result of applying the described approach, we obtain a descriptive fuzzy model of bank competitiveness, and also characteristics of quality for obtained statements. Using max-min convolution transform of fuzzy values for each complex sign in program *FExcel* and its normalization by formula (19), we will get a final evaluation of investigated condition for bank “Khreshchatyk” competitiveness (2015 year).

The value of the normalized generalizing index $I = f_Y(X_1, X_2, X_3, X_4, X_5)$ is shown in Table 5. Thus, bank “Khreshchatyk” competitiveness (for the 2015 year), which is calculated by using the proposed fuzzy technique, was on an average level in comparison with competitiveness of this bank in 2007-2011 years.

Table 5: The value of the fuzzy generalizing index *i* for bank "Khreshchatyk" competitiveness (2015 year).

Generalized values for groups of indicators					Weight	Output variable
X_1	X_2	X_3	X_4	X_5	ω	I
0,617	0,472	0,301	0,536	0,439		0,613
H	*VH*	*H*	*M*	*M*	ω^{I}_{11}	*M*

It is interesting to compare our results with the results of other authors (using other methods). So, the research results for the study of relative technical, pure technical and scale efficiency of Ukrainian banks activities for the period from 2005 to the 2015 year by using nonparametric DEA (Data Envelopment Analysis) approach, and on the basis of analysis information which is published quarterly on the official website of National Bank of Ukraine, were given in (Dolgikh, 2016). Most indicators of bank effectiveness coincide with bank competitiveness indicators. «Khreschatyk» bank had a value 0.522 of relative effectiveness in 2015. Considering that banks effectiveness is proposed to measure in the range [0, 1], value 0.522 corresponds to an average value for an indicator of effectiveness, which fully confirms the value obtained for generalizing parameter of competitiveness for "Khreschatyk" bank in 2015.

Thus, a methodological approach for development generalizing parameter of bank competitiveness using the tools of fuzzy sets has been proposed. It together takes into account crisp and fuzzy data, in other words, metric and non-metric values under uncertainty. It causes objectivity and adequacy of determining real level of bank competitiveness and, therefore, the effectiveness of management decisions concerning bank functioning and development, adopted on the basis of analysis and evaluation of its level.

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