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Banknote Demand in the European Union: Different Seasonal Patterns

Abstract: A model based on evolving splines is proposed to address seasonal patterns in daily cash demand observed in banknotes issued by the national central banks of Spain and Germany. Statistical indexes are applied to measure and compare weekly, monthly and yearly seasonal variations in these two series. Changes in these seasonal patterns are identified. The main finding is that seasonal variations are less relevant in the German series. By contrast, in the Spanish case, the magnitude of seasonal variations is increasing. Therefore, high levels of dissimilarity are observed between both cases for each one of the three seasonal patterns. However, the lower values of complementarity indexes suggest that the shapes of these seasonal patterns are not so different.

Key words: banknote issues, European Union, seasonal variations, splines

JEL Classification: C22, E41

1. Introduction

The demand for money plays a key role in the efficient functioning of modern economic systems. Therefore, monetary authorities manage money supply and interest rates as some of the tools to achieve their aims. Such authorities should take any increase in money demand into account to provide sufficient liquidity according to demand from economic agents (Galán Figueroa & Venegas Martínez, 2016). In general, national central banks provide cash to commercial banks who need to satisfy their clients' demands, whether companies or individual customers. In the particular case of the euro area, the monetary authority is the European System of Central Banks (ESCB), consisting of the European Central Bank and the national central banks of the member states whose currency is the euro. According to Article 127 of the Treaty on the Functioning of the European Union (Official Journal of the European Union, 59, 2016/C 202/01), the ESCB's primary objective is to maintain price stability. To this end, one of the basic tasks to be carried out through the ESCB is to define and implement the monetary policy of the Union.

As the recent movements in interest rates have shown in the last few years, this instrument seems to be the most frequently applied to control inflation, whereas euro banknotes are issued on the initiative of commercial banks. As a result of the Decision of the European Central Bank (ECB) of 6 December 2001 on the euro banknotes issue (ECB/2001/15, paragraph 3), the issue of banknotes need not be subject to quantitative or other limits, since putting banknotes into circulation is a demand driven process in the way that the volume of euro banknotes produced annually must be enough to cover the annual cash demand. Furthermore, stocks must always be available in sufficient quantity to cover seasonal fluctuations. To estimate the annual cash demand, the national central banks calculate their national demand for banknotes for each year and submit the results to the ECB. The ECB then estimates the demand for the Eurosystem as a whole and, according to national populations and gross domestic products of member states, each national central bank in the euro area is assigned a share of the overall annual production volume for euro banknotes (Schautzer, 2007)[1]. Subsequently, the total value of euro banknotes issued by the central banks in the Eurosystem is distributed among these banks on the last business day of each month in accordance with the official banknote allocation key. The mismatch between these shares and the actual issues by national central banks is shown in their balances as a liability item, if the actual issue is more than the allocated amount, or as an asset item, if the actual issue is below the allocated amount. In addition, national central banks may need to compensate imbalances in cash circulation with cross-border cash transports on an ad-hoc basis according to demand[2].

1 The European Central Bank and the national central banks of participating countries have the exclusive right to issue euro banknotes (Decision of the European Central Bank of 6 December 2001 on the issue of euro banknotes, ECB/2001/15, paragraph 1). In practice, only the national central banks put euro banknotes into circulation and later withdraw them from circulation.

2 As explained by Schautzer (2007), these cross-border flows result in national imbalances in cash circulation which cannot be accounted for completely in banknote production and distribution planning. Such flows are driven by several factors, including economic ties between member states, cross-border commuting, labour migration, tourism or the tendency to hoard cash, influenced by aspects such as the preferences for specific payment methods.

The volume of banknotes in circulation indicates the value of outstanding banknotes issued by the Eurosystem on a given date. In statistical terms, the volume of banknotes in circulation is derived from the cumulative difference, in terms of value, of the banknotes paid out by the central bank and those deposited with it. In other words, such a value is the cumulative difference of the value of banknotes issued by the national central banks and the value of banknote withdrawals by these banks. Furthermore, cash migrates freely within the euro area. Therefore, as explained in Bundesbank 2015 Annual Report (Deutsche Bundesbank, 2015), the volume of banknotes in circulation issued by a single national central bank does not initially say anything about how much cash is actually in circulation in the issuing country.

From the point of view of member states, the issue of banknotes should adjust to the banknote demand. This demand can be broken down into transaction balances (banknotes held by enterprises and consumers to conduct transactions), hoarding (banknotes serve as a store of value) and foreign demand (transaction balances and hoarding outside the country of issue). Of course, cash demand caused by transaction motivation or by hoarding is influenced by factors like the level of economic development or the interest rate, but these demands are also affected by the spatial distribution of bank services and the payment technologies[3]. On the other hand, a portion of cash demand is related to the level of underground economy or even the existence of illegal activities (Pickhardt & Sarda, 2011, 2015; Ardizzi et al., 2014; Bartzsch & Seitz, 2016; Rua, 2018)[4]. Therefore, the differences in the behaviour of economic agents participating both in the formal economy and in the shadow economy, as well as the different relative participation of these two economic sectors, are expected to cause different patterns of net issues by the national central banks of EU member states.

Given the German share in the total value of euro banknotes issued in the euro area, the Bundesbank plays an important role in the Eurosystem. The volume of German euro banknotes in circulation has increased much more than the circulation of banknotes issued by other Eurosystem member states. However, the domestic demand for banknotes does not make a notable contribution to this growth, which can be primarily explained by the volume of German-issued euro banknotes held abroad and, to a large extent, driven by demand from outside the euro area (Bartzsch et al., 2013; Bartzsch & Seitz, 2016). Therefore, the evolution of banknotes issued by the Bundesbank is noticeably different from the one cor-

[3] The relative importance of different payment methods varies among countries in the European Union (Bagnall et al., 2016; Kalckreuth et al., 2014).

[4] Rainone (2023) shows that policy measures to fight tax evasion can affect the demand for cash.

responding to other national central banks like the Bank of Spain or the Bank of Portugal.

In the Spanish case, cash demand increased noticeably during the COVID-19 pandemic, in such a way that the net issue of banknotes, usually negative before the pandemic, became positive in 2020. However, it seems to be returning to the previous trend, although 2022 also ended with a negative net issue. During the pandemic period, an increase of cash demand as a store of value was observed whereas their use as a means of payment diminished[5]. Furthermore, these changes were also affected by the restrictions to international travel, which caused the interruption of tourism flows and the resulting entry flows of banknotes in Spain. The relevance of this banknotes flow related to tourism activity is also a factor to explain the evolution of net issues by the central bank of Portugal (Rua, 2018).

Apart from different trends in national net issues, the relative participation of domestic or external cash demand also causes differences in seasonal patterns. For example, external cash demand covered with banknotes issued in Germany is characterised by a weak or null seasonality. Especially the demand from outside the euro area is more related to the international reputation of the euro and, therefore, even though there is a seasonal variation in domestic cash demand, the total value of cash in circulation exhibits a dampened seasonal behaviour (Bartzsch et al., 2013). Nevertheless, despite the high participation of net issues by the German central bank, daily series of banknotes in circulation in the euro area also exhibits seasonal variations that show periodic patterns within a week, a month or a year[6]. Furthermore, these seasonal variations are changing. For example, the widespread use of automated teller machines and credit or debit cards may be making the seasonal fluctuations weaker[7].

[5] In the context of the pandemic crisis, central banks were swift to disseminate information regarding the low probability of infection through banknotes. Conversely, restrictions to mobility had impact an impact on the increase of e-commerce and the use of cashless means of payments, which have contributed to a decline in the demand for cash transactions. The effect of pandemic on banknote demand are analysed, among others, by Kotkowski (2023), Skibinska-Fabrowska (2023) and Wisniewski et al. (2024).

[6] As explained in Cabrero et al. (2009), the number of banknotes in circulation increases just before the weekend and decreases after the weekend, decreases before the middle of the month and increases towards the end as a result of the payment of salaries and rises during the summer holidays and towards the end of the year. Ollech (2018) also indicates that this daily time series in the German case contains three different seasonalities: the demand for banknotes increases towards the weekend, salary payments tend to be concentrated around the turn of the month and the demand reaches its peak around Christmas time.

[7] Cabrero et al. (2009) comment that the impact of automatic teller machines may change the weekly seasonal pattern.

The aim of this paper is to illustrate the differences in seasonal patterns of daily cash demand observed in banknotes issued by the national central banks of Spain and Germany and detect changes in these seasonal patterns. Modelling these patterns is not a simple task. When the length of the seasonal period increases, there are difficulties to apply traditional seasonal ARIMA or structural time series models. Therefore, modelling seasonal variations in terms of trigonometric terms as proposed by De Livera et al. (2011) or by using spline functions in structural time series models, as suggested by Harvey et al. (1997), are interesting proposals. However, some problems remain to deal with a non-fixed length of the seasonal period. Cabrero et al. (2009) propose to use missing values to accommodate time series models to the changing number of days in a year, a month or even a week. Ollech (2018) also indicates that the length of these seasonal cycles must be standardised, either by omitting a subset of the data or by artificial prolongation. In this sense, we suggest an alternative solution by defining the seasonal effect at a season as a function of the proportion of the seasonal period elapsed up to the season and applying a restricted evolving spline model. Furthermore, and considering that comparisons of seasonal patterns of banknotes are useful to estimate the share of transactional or non-transactional demand[8], an added value of this paper consists of the application of dissimilarity and complementarity indexes to compare seasonal patterns.

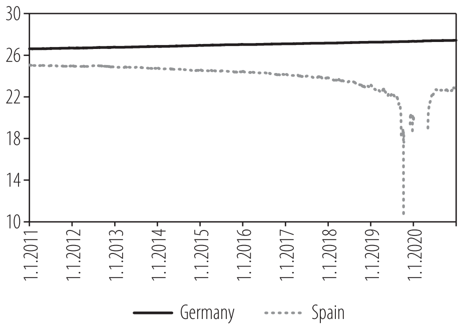
2. Material and methods

2.1. Net issue of banknotes in the euro area

Figures are not included to save space, but the evolution of the stock of net issues at the end of month by the national banks of Germany, France, Italy and Spain show that these are the member states with a major share of the overall annual production volume for euro banknotes and in the total value of euro banknotes issued in the euro area. The relative importance of the value of banknotes issued by the Bundesbank is noticeably higher than its key allocation proposed by the Eurosystem. By contrast, actual net issues in Spain are lower than the corresponding participation according to its key allocation. Furthermore, there has been an upward trend in Germany, as well as a downward trend in Spain in euro banknotes issues after the economic crisis in 2008.

[8] To estimate the share of non-transactional demand in Japan, Otani & Suzuki (2008) make a comparison of seasonal pattern of a series of low denomination banknotes, assumed to be held for transaction demand, with the corresponding seasonal pattern of a series of high denomination banknotes, assumed to be held for both transaction and non-transaction demand.

Graph 1: Net issues of euro banknotes by the German and Spanish national banks (daily series, natural logarithms, 2011-2020).



Source: Deutsche Bundesbank, Bank of Spain

An interesting question addresses the hypothesis that the explanatory factors of cash demand explaining these different trends can also help explain different yearly seasonal patterns. Furthermore, seasonal variations within a month or a week are not possible to detect in monthly series. To analyse these seasonal cycles in a more precise way, daily data about net issues from the 1st of January 2011 to the 31st of December 2020 were provided by both the Spanish central bank and Deutsche Bundesbank. Graph 1 shows these figures in natural logarithms. Note that, especially in the Spanish case, there are several anomalous observations in 2020 related to the pandemic period[9].

2.2. Methodology

Let $\{y_t\}_{t=1,\dots,T}$ be a daily series such that the observed value in day t, y_t, is assumed to be decomposed into three different latent time varying components: a trend component, representing an underlying level; a complex seasonal component, with multiple seasonal cycles of different lengths; and, finally, a remainder or residual component, as follows:

$$y_t = \mu_t + \gamma_t + \varepsilon_t, t = 1, \dots, T \quad (1)$$

where μ_t, γ_t and ε_t are, respectively, the trend component, the seasonal component and an error term in day t. The seasonal component is assumed to be the sum of three types of seasonal cycles (yearly, monthly and weekly), that is to say, $\gamma_t = \gamma_t^{year} + \gamma_t^{month} + \gamma_t^{week}$, where γ_t^{sc}, with $sc = year, month, week$, represents each of these three seasonal cycles when the period of the cycle is a year, a month or a week, respectively.

[9] Note that the accumulated net issue at the end of month in Spain turned negative for the first time in September 2019. Spaces in the line in Graph 1 correspond precisely to negative observations at levels in the original series, which were observed over many days in the period between September 2019 and April 2020.

Note that net issue is not registered every day of a year, so that the number of days in a week changes, as does the number of observations in a month or a year. Taking these changes in the length of seasonal periods into account and following the proposal by Martin-Rodriguez & Caceres-Hernandez (2010), the seasonal effect at a season is defined as a function of the proportion of the length of the seasonal period elapsed up to this season. To do this, the value of one of these seasonal variations on day t, γ_t^{sc}, can be defined as a function of both the specific seasonal cycle to which day t corresponds and the proportion of the length of its seasonal period elapsed up to the season corresponding to that day. Therefore, when this seasonal cycle is observed m times over the sample period, and day t corresponds to seasonal cycle c, $c = 1, \dots, m$, whose length is l_c, and, specifically, to season j_c, $j_c = 1, \dots, l_c$, then the value of this seasonal variation on day t can be defined as $\gamma_t^{sc} = \gamma_{c,w}$, where $w = \frac{j_c}{l_c}$, and, therefore, $0 < w \leq 1$.

Each seasonal variation is modelled by using a restricted evolving spline model (RESM) following the proposal in Caceres-Hernandez & Martin-Rodriguez (2017). Estimates of seasonal effects in a seasonal cycle are obtained from a continuous periodic spline function restricted to the integral being equal to zero over the whole seasonal period. Thus, the integral of the absolute value of the spline function over this seasonal period is a measure of the relevance of the seasonal variation. Therefore, the relative importance of simultaneous seasonal variations can be evaluated, and the relative magnitude of seasonal variations within the same seasonal period for different series can also be compared.

2.2.1. Restricted evolving spline model

The model is obtained in three steps[10]. First, estimates of the value of seasonal effect at every proportion w on the continuous interval [0,1], corresponding to the length of the seasonal period, are obtained from a periodic cubic spline, i.e., a restricted piecewise third-degree polynomial function of such proportion (see Appendix, Graph A.1.a). The seasonal variation at any proportion w in a seasonal cycle can also be expressed as a linear function of the values of seasonal effects at specific proportions (break points) of the period in the seasonal cycle as shown in Graph A.1.b (Appendix). Note that the parameters of the spline adjusted to each seasonal cycle over the sample period are not restricted to being fixed. In this way, changes in both the magnitude of seasonal effects and the shape of the whole seasonal pattern can be captured. In fact, changes in the shape of the seasonal

[10] The details are explained in Caceres-Hernandez and Martin-Rodriguez (2017). See also Caceres-Hernandez et al. (2022) and Martin-Rodriguez and Caceres-Hernandez (2023).

pattern over time can be explained by describing the evolution of the seasonal effects at these specific proportions.

With this aim, in a second step, a non-periodic cubic spline is adjusted to the values of seasonal effects at a specific break point over time. Once the estimates of the values of seasonal effects at a specific break point for each of the m cycles over the sample are obtained, new estimates of the seasonal effect at this break point for the seasonal cycle c are obtained from a non-periodic cubic spline of proportion $w_c = \frac{c}{m}$ (Appendix, Graph A.2.a), i.e., the proportion of the number of seasonal cycles over the sample elapsed up to the seasonal cycle c. Then, the values of seasonal effects at this break point for each seasonal cycle can be expressed as a linear function of the seasonal effects at the same break point for some of the seasonal cycles in the sample (Appendix, Graph A.2.b). Note that forecasts for seasonal effects at this break point of the seasonal period in future seasonal cycles can be obtained from these functions. The same procedure is applied to seasonal effects at each of the break points selected in the first step.

Finally, the evolving seasonal pattern is expressed as a function of the values of seasonal effects at the selected break points of the seasonal period in the chosen seasonal cycles as shown in Graph A.3 (Appendix). Subsequently, forecasts of the whole seasonal pattern h seasonal cycles ahead can be obtained from the previous forecasts of seasonal effects at the break points in these seasonal cycles (see also Appendix, Graph A.3).

2.2.2. Measuring and comparing simultaneous seasonal variations

The relative importance of simultaneous seasonal variations can be evaluated by measuring the integral of the absolute value of the spline function over the whole seasonal period. Therefore, the relative magnitudes of seasonal variations within the same seasonal period for different series can also be compared. Furthermore, for seasonal variations with the same seasonal period in different series, the dissimilarity and complementarity between these variations can be assessed by calculating the area between the corresponding spline functions for the first case and calculating the area in which both spline functions have different signs for the second case. These statistical indexes are also explained in Caceres-Hernandez et al. (2022) and Martin-Rodriguez & Caceres-Hernandez (2023).

From the results of estimating the corresponding RESM for all the seasonal variations (yearly, monthly and weekly), the estimates of each seasonal variation can be obtained. Let $\gamma_{c,w}^{sc}$ be the estimates of the seasonal effect for the seasonal variation at proportion w in the seasonal cycle c, where $sc = year, month, week$, corresponds to the yearly, monthly and weekly seasonal patterns, respectively. Then, once the proportions in the seasonal cycle c at which the spline function is equal to zero are obtained, the magnitude of the seasonal variation in the whole seasonal period, A_c^{sc}, can be estimated as shown in Graph A.4 (Appendix). Thus, from calculating the areas A_c^{year}, A_c^{month}, and A_c^{week}, the relative magnitudes of these different simultaneous seasonal variations can be evaluated[11].

On the other hand, let $A_c^{sc,X}$ and $A_c^{sc,Y}$ be the corresponding areas for the seasonal variation in the same seasonal cycle for two different time series, calculated as shown in Graph A.4 (Appendix). A measure of the differences between these seasonal variations, $DA_c^{sc,X,Y}$, can be obtained by assessing the magnitude of the area between the spline functions corresponding to the compared seasonal patterns (Appendix, Graph A.5.a),

$$DA_c^{sc,X,Y} = \sum_{r_{X,Y}=1}^{l_{X,Y}} \left| \int_{w^c_{r_{X,Y-1}}}^{w^c_{r_{X,Y}}} \left(g_c^{sc,X}(w) - g_c^{sc,Y}(w) \right) dw \right|, \tag{2}$$

where $g_c^{sc,X}$ and $g_c^{sc,Y}$ are, respectively, the evolving spline functions estimated for series X and Y, and $\{w^c_{r_{X,Y}}\}_{r_{X,Y}=1,\dots,l_{X,Y}}$ are the ordered values of proportions w in the seasonal cycle at which $g_c^{sc,X}(w) = g_c^{sc,Y}(w)$, and also $w_0^c = 0$, and $w^c_{l_{X,Y}} = 1$. Thus, a dissimilarity index can be defined as

$$DI_c^{sc,X,Y} = \frac{DA_c^{sc,X,Y}}{A_c^{sc,X} + A_c^{sc,Y}}. \tag{3}$$

However, when seasonal effects at specific proportions of the length of the seasonal period have opposite signs for both series, there is a level of complementarity between seasonal variations. The degree of complementarity can be assessed from calculating the magnitude of the area, $CA_c^{sc,X,Y}$, between the spline functions corresponding to the compared seasonal patterns over the intervals inside the seasonal period when both spline functions have different signs (Appendix, Graph A.5.b),

$$CA_c^{sc,X,Y} = \sum_{r*_{X,Y}=1}^{l*_{X,Y}} \left| \int_{w^c_{r*c_{X,Y-1}}}^{w^c_{r*_{X,Y}}} \left(g_c^{sc,X}(w) - g_c^{sc,Y}(w) \right) dw \right|, \tag{4}$$

[11] The idea is similar to the proposal in Martin-Rodriguez & Caceres-Hernandez (2013) to measure the evolution of yearly seasonal patterns in weekly series. However, to compare the magnitude of seasonal variations with different seasonal periods, the areas calculated according to Graph A.4 (Appendix) to measure the relevance of weekly, monthly and yearly seasonal variations in a day should be multiplied by the number of days in each seasonal period.

where $\{w^c_{r*_{X,Y}}\}_{r*_{X,Y}=1,\dots,l*_{X,Y}}$ are the extreme values of the contiguous intervals obtained by ordering the values of the sets $\{w^c_{r_X}\}_{r_X=1,\dots,l_X}$ and $\{w^c_{r_Y}\}_{r_Y=1,\dots,l_Y}$ in which the sign of $g^{sc,X}_c(w)$ is opposite to the sign of $g^{sc,Y}_c(w)$, and also $w^c_0 = 0$, and $w^c_{l*_{X,Y}} = 1$. Thus, a complementarity index can be defined as

$$CI^{sc,X,Y}_c = \frac{CA^{sc,X,Y}_c}{A^{sc,X}_c + A^{sc,Y}_c}\,. \tag{5}$$

3. Results

The RESM model and indexes proposed in the previous section are useful tools to identify which and how different seasonal components in the daily series of net issues are changing and how the locations of peaks and troughs evolve. It was decided to estimate seasonal patterns in the period between 2011 and 2018, whereas 2019 is assigned as a forecasting period to assess up to what point the data generating process deviates from that observed until 2018[12].

As commented, the role of Deutsche Bundesbank in the euro area and, above all, the external demand suggests different seasonal patterns compared to the Spanish case, with important effects of economic activities like tourism. The preferences for different payment methods could also contribute to changes in these seasonal patterns. Therefore, it is interesting to detect such changes and compare the magnitude of seasonal variations in both cases.

3.1. Estimating the seasonal model

For both series, daily net issue is assumed to be an additive combination of level, seasonal, and noise components. The underlying levels in Graph 1 do not seem to be fixed, whereas a complex seasonal pattern around a stochastic trend may exist. Therefore, an unobserved components model was formulated as

$$y_t = \mu_t + \gamma^{year}_t + \gamma^{month}_t + \gamma^{week}_t + \textstyle\sum_{date} \beta_{date}\, D^{date}_t + \varepsilon_t \tag{6}$$

[12] Net issues are usually not registered on Saturdays, Sundays and other festive days. The number of observations between 2011 and 2018 was 2,024 for the series provided by the Bank of Spain and 2,102 for the series provided by Deutsche Bundesbank, whereas net issues were registered in 254 and 256 different dates in 2019, respectively. Data corresponding to 2020 are also available but have not been used to assess the forecasting performance of the models due to the expected impact of COVID-19.

where y_t is the natural logarithm of the net issues, μ_t is a local level trend component with fixed slope (see Harvey 1989, pp. 37-38), γ_t^{year}, γ_t^{month} and γ_t^{week} are the yearly, monthly and weekly seasonal variations, respectively, and ε_t is a disturbance term. Three different restricted evolving splines are included to capture yearly, monthly and weekly seasonal cycles[13].

From calculating centred moving averages, preliminary estimates of the magnitude of seasonal cycles are obtained. The first estimates for the weekly cycle are obtained by subtracting a 3 to 7-term moving average from the original series in logarithms according to the number of days corresponding to the specific week. To obtain estimates of the monthly cycle, moving averages are calculated from the previous weekly moving averages by using a number of terms depending on the number of days corresponding to the specific month. Finally, from the monthly moving averages, the yearly cycle is estimated by calculating other moving averages with a changing number of terms according to the number of days with data registered in the year[14]. These preliminary estimates suggest that weekly, monthly and yearly seasonal cycles do not seem to be fixed across the sample. Therefore, evolving splines are a sufficiently flexible model to capture these features[15]. Graph 2 shows the estimates of a fixed seasonal pattern adjusted to the initial estimates of the structural model formulation for each seasonal cycle. Note that seasonal variations are more relevant in Spain, moreover, the yearly seasonal pattern seems to be more noticeable than the monthly or weekly seasonal variations.

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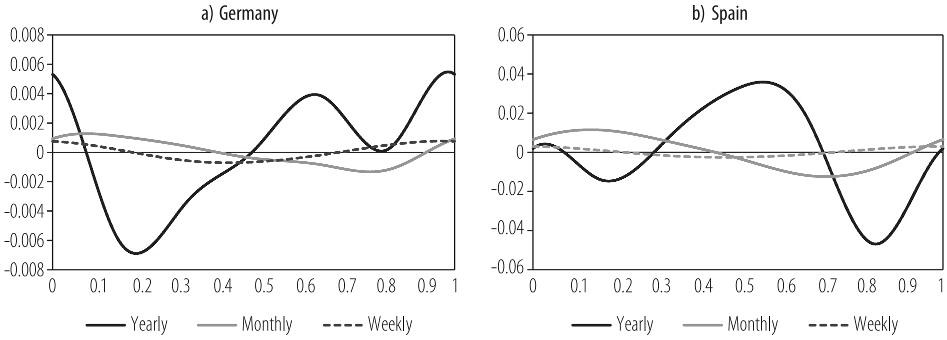
[13] Note that the length of the seasonal period in weekly cycles is not fixed. Therefore, in spite of the short length of the seasonal period, spline functions are advantageous to model this type of seasonal variations. Dummy variables were also included as exogenous regressors to capture anomalous observations. However, the estimates suggest that this type of observations is not present.

[14] According to data registered by the central banks for the period 2011-2019, the length of the seasonal period in weekly cycles changes from 4 to 7 for the German series and from 3 to 5 for the Spanish series. In monthly cycles, the number of days oscillates from 20 to 27 for the German series and from 18 to 23 for the Spanish series. Finally, the length of the yearly seasonal cycles moves from 260 to 267 in the case of Germany and from 251 to 254 in the Spanish case.

[15] Three-segment cubic splines with equally spaced break points were applied to weekly seasonal cycles, whereas six-segment cubic splines were chosen for both monthly and yearly seasonal cycles.

Graph 2: Fixed seasonal effects



According to the estimates of yearly seasonal effects for Germany, maximum values of net issues are registered near the end of the year, whereas minimum values are between the end of the winter and the beginning of the spring season. Then, an upward movement leads to a secondary peak at the end of summer. The estimates for Spain suggest more clearly that the highest positive effects correspond to the summer period. However, monthly and weekly seasonal effects seem to be more similar in both series. Although the presence of these two seasonal cycles is not so clear, these seasonal patterns may also be changing. To check this hypothesis, three seasonal variations are assumed to be present: yearly, monthly and weekly effects.

The results from estimating the corresponding models are included in Table 1[16]. The estimates of the stochastic levels are clearly higher for Germany. However, the main features are changes in the magnitude and shape of seasonal cycles over the sample period. The fitting sample is 2011-2018 and the holdout sample is 2019 in such a way that the end of 2018 is the forecast origin to generate forecasts for trends and each of the seasonal cycles for any day in 2019[17]. The noticeable decrease in net issues by the Spanish central bank in 2019, with negative values from September, contribute to explaining the mismatch between forecast and observed values shown in Graph 3.

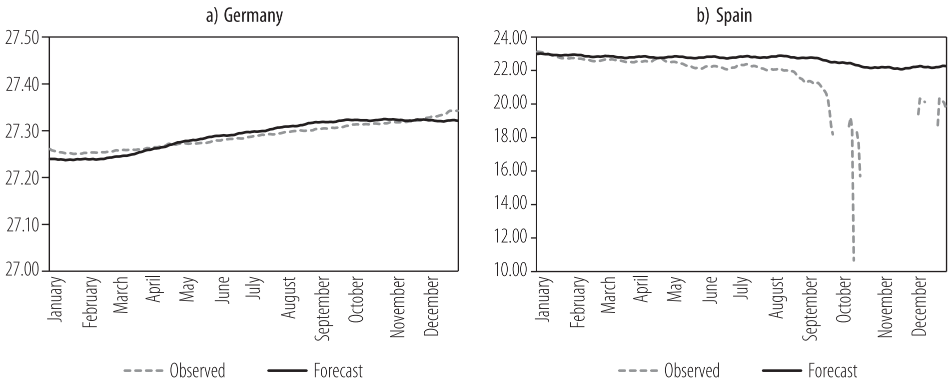
[16] Estimates by maximum likelihood are obtained by using the Stamp 8.30 module of Ox-Metrics 6.20 package.

[17] The forecasts for the seasonal cycles were obtained as shown in Graph A.3 (Appendix). Forecasts of daily net issues were obtained from these forecasts for each seasonal component and for the level component (by using a non-periodic spline adjusted to the estimates of the stochastic level from 2011 to 2018).

Table 1: Results of estimating structural time series models (2011-2018)

	Germany	Spain
Log-likelihood	2,081.22	2,052.07
-2logL	-4,162.43	-4,104.13
Prediction error variance	0.115736	0.109723
Information criterion Akaike (AIC)	-2.0737	-2.1238
Bayesian Schwartz (BIC)	-1.8398	-1.8826
Standard deviations of disturbances		
Level	0.208289	0.202973
Slope	0.00675779	0.00658531
Irregular	0.213700	0.208246

Figure 3: Forecast values (2019)

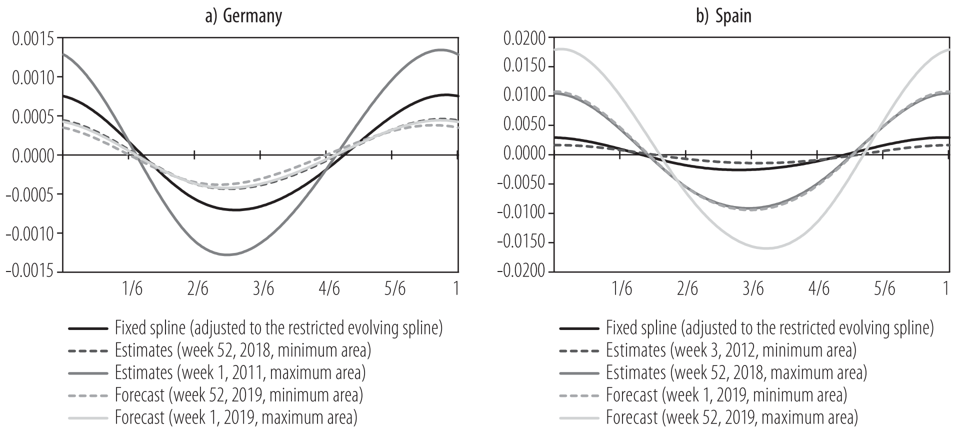


3.2. Description of seasonal effects over the sample period

Graphs 4 to 6 show the estimates of the different seasonal cycles. It is worth noting both the shape of the seasonal patterns and the changes in the magnitudes of the seasonal effects corresponding to monthly and weekly seasonal cycles. Graphs 4 and 5 include: a) the estimates of a fixed seasonal pattern adjusted to the estimates of seasonal effects according to the RESM model over the fitting sample period, and b) the estimates of seasonal effects according to the RESM model corresponding to the seasonal cycle (month or week) in which the magnitudes of the seasonal variation are minimum and maximum. These magnitudes are measured in terms of the areas shown in Graph A.4 (Appendix). To observe the changes in the yearly seasonal cycle, Graph 6 shows the estimates from the RESM model and the estimates of the fixed seasonal pattern adjusted to these evolving seasonal effects. In a similar way, the forecasts of seasonal cycles are compared in Graphs 4 to 6, with fixed seasonal patterns over the sample period.

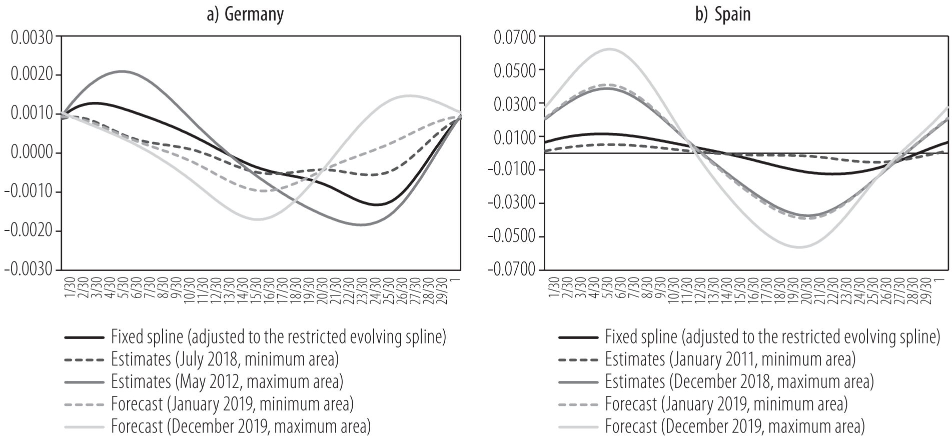
Note that the troughs in the weekly seasonal pattern are located around the middle of the week. However, the changing length of the weekly seasonal period makes it difficult to identify a specific day of the week with minimum values of net issues. This length tends to be shorter in Spain, but the magnitude of seasonal effects is noticeably higher. Furthermore, according to the areas measuring the relevance of this seasonal variation, the behaviour differs clearly between Germany (decreasing) and Spain (increasing). In fact, the most relevant weekly seasonal variation corresponds to the first week in 2011 in Germany and to the last week in 2019 in Spain.

Graph 4: Estimates and forecasts of weekly seasonal effects

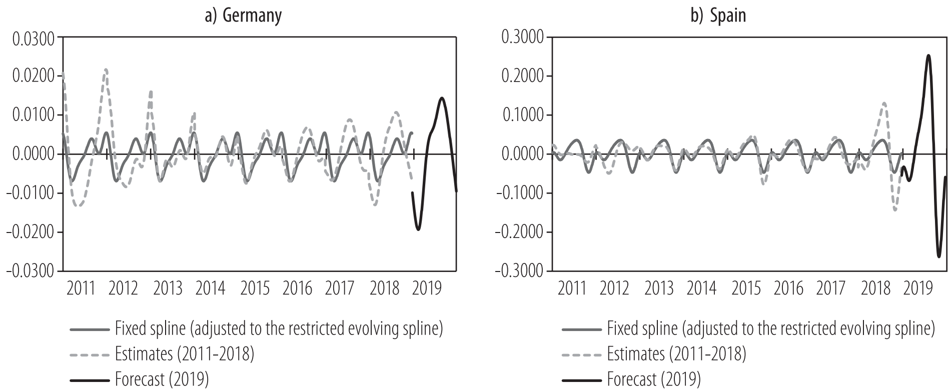


Monthly seasonal effects seem to move in a range with more amplitude than weekly seasonal effects. In Germany, changes in the monthly seasonal pattern over the sample period suggest the location of troughs is moving from the second part of the month to the middle, whereas the location of peaks is moving from the beginning to the end of the month. In the Spanish case, locations of peaks and troughs seem to be more stable, but the magnitude of seasonal effects is clearly increasing, as suggested by the comparison of seasonal pattern for January 2011 with the one for December 2019. In the last seasonal pattern of the sample, the peak is located around the first quarter of the month, whereas the trough corresponds to the second part of the month.

Graph 5: Estimates and forecasts of monthly seasonal effects



Graph 6: Estimates and forecasts of yearly seasonal effects

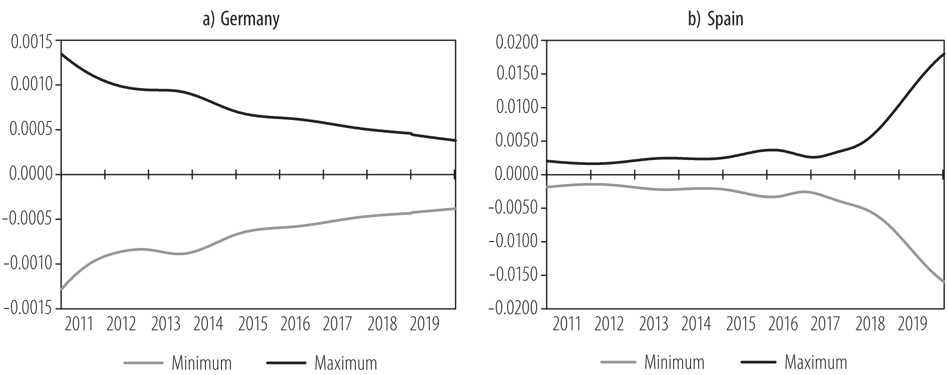


Noticeable differences are also observed in yearly seasonal patterns. Although the magnitude of positive and negative seasonal effects is smaller in Germany, an upward trend in these magnitudes is observed according to the estimates for the last years in the sample-period and, therefore, the forecasts of seasonal effects in 2019 are higher than in previous years. Furthermore, whereas during the first years in the sample the peaks occur at the beginning and the end of the year, in the later years, they shift closer to the middle of the year. In Spain, peaks are usually located before the summer period, but they seem to be moving to the summer period, as occurred in 2018 and in 2019, according to the forecasts of seasonal effects.

In each of the three seasonal cycles, the fixed splines suggest the corresponding seasonal variations are less relevant than suggested from the estimates obtained by allowing these seasonal effects to evolve over the sample period. In this sense, to show the changes in these seasonal patterns, the RESM model is more useful than other alternative models assuming fixed seasonal patterns.

The extreme values of the seasonal effects also provide a useful description of the evolution of seasonal patterns. As shown in Graph 7, peaks and troughs in weekly seasonal cycles become closer in Germany, whereas the distance between them increases in Spain. Such an increase is especially clear from the first weeks in 2018, when the absolute values of both negative and positive seasonal effects tend to increase. Therefore, the relevance of this seasonal cycle seems to decrease in Germany and yet increases in Spain.

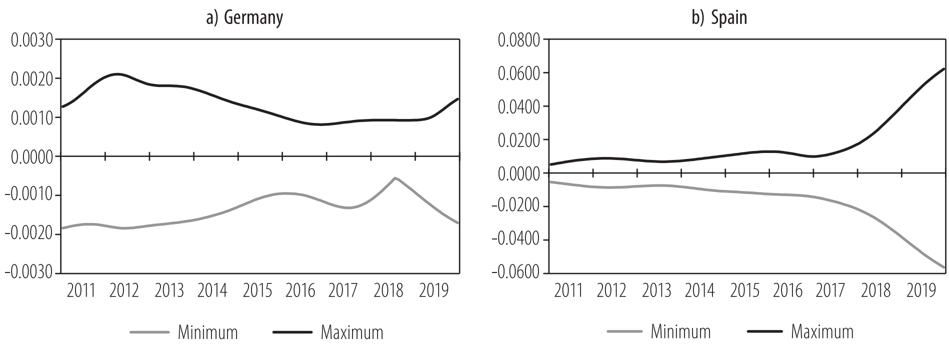
Graph 7: Evolution of the extreme values of the weekly seasonal effects



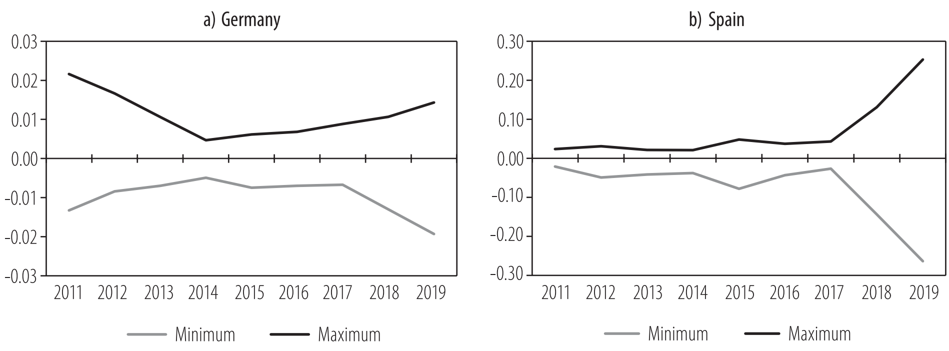
Similar behaviour is observed in monthly cycles (Graph 8), although the distance between peaks and troughs in Germany does not decrease so clearly. In fact, the relevance of this seasonal cycle seems to increase from the middle of 2018. With regard to the yearly seasonal cycle in Germany (Graph 9), the distance between peaks and troughs decreases from the beginning of the sample until 2014, but the relevance of this seasonal cycle seems to increase after that year and especially from 2017. In Spain, maximum and minimum seasonal effects are more stable until 2016, but from 2017 to 2019 these values are more distant, which also suggests the yearly seasonal variation is becoming more important.

According to the evolution of extreme values throughout the sample, weekly, monthly and yearly seasonal variation are less relevant in Germany than in Spain. Furthermore, the relevance of weekly seasonal cycles is clearly decreasing in Germany, whereas the relevance of weekly, monthly and yearly seasonal cycles is increasing in Spain from 2017. A conclusion regarding the evolution of the relevance of these seasonal patterns over time is best obtained from the assessment of areas proposed in the methodological section.

Graph 8: Evolution of the extreme values of the monthly seasonal effects



Graph 9: Evolution of the extreme values of the yearly seasonal effects



3.3. Comparison between simultaneous seasonal variations in a time series

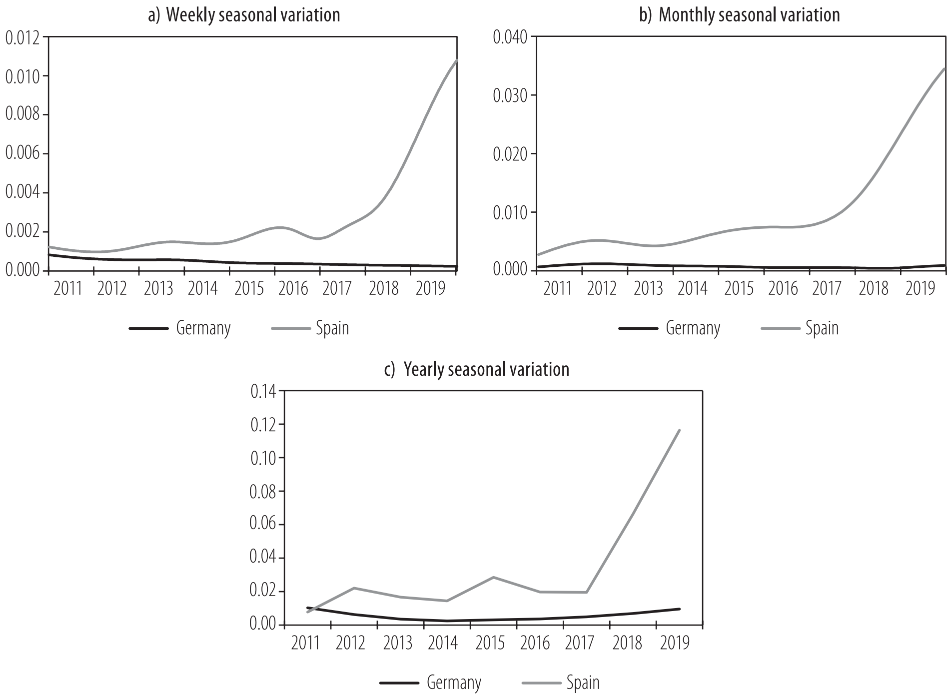
The indexes proposed in the methodological section are useful tools to compare different seasonal cycles and to compare the same seasonal cycles in Germany and in Spain.

Graph 10 shows the evolution of the magnitude of the weekly, monthly and yearly seasonal cycles, measured in terms of the integral of the absolute value of the corresponding spline function as shown in Graph A.4 (Appendix). As previously commented, all three seasonal cycles are less relevant in Germany. Although there are changes in the magnitude of these seasonal variations, the most salient feature is the noticeable upward trend from 2017 for each seasonal cycle in the Spanish case.

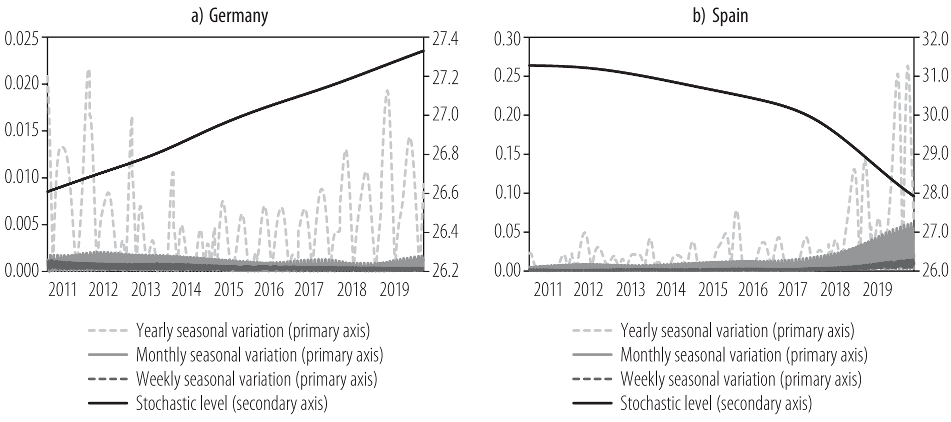
However, these areas are calculated by considering the seasonal period to be the interval (0,1), despite this period corresponding to between 3 and 6 days for weekly cycles, 18 to 27 days for monthly cycles and, finally, 251 to 267 days for yearly cycles. To capture the relative relevance of different seasonal cycles at specific daily observations, weekly, monthly and yearly areas are adjusted to a daily basis, first by calculating the integral of the absolute value of the spline for the part of the seasonal period corresponding to a day, and second by multiplying the resulting area by the number of days in a complete seasonal period. The results of these adjustments are shown in Graph 11 and include the integral of a non-periodic six-segment spline adjusted to the estimates of the level component. Given that the spline is defined as a function of the proportion of the fitting sample period (2,102 days for Germany series and 2,024 days for Spain series), the area corresponding to each day from 2009 to 2019 is multiplied by 2,102 or 2,024 to enable a comparison with the areas corresponding to the seasonal cycles.

Although a different scale is applied to the secondary vertical axis to represent the areas corresponding to the level, the values of adjusted areas for this component are always clearly higher than the adjusted areas for seasonal cycles. However, it is clearly observed that the yearly seasonal cycle is the most important seasonal variation for most days both in Germany and in Spain. Furthermore, the monthly seasonal cycle is more relevant than the weekly cycle.

Graph 10: Magnitudes of the weekly, monthly and yearly seasonal variations (integral of the absolute value of splines)



Graph 11: Magnitudes of trend and seasonal variations (adjusted integral of the absolute value of splines, daily basis)



3.4. Comparison of the seasonal variation corresponding to a seasonal cycle between the two series

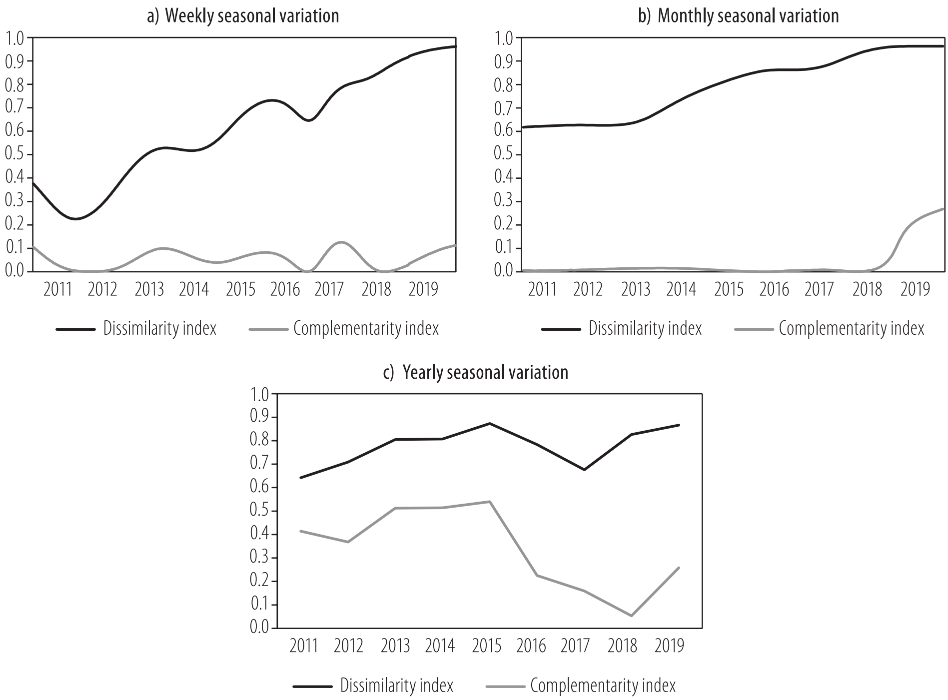
The magnitude of extreme values of seasonal effects and the location of points in the seasonal period where the sign of seasonal effects changes are the changing features of yearly, monthly and weekly seasonal patterns in net banknote issues. As a result of these changes, the shape and relevance of these seasonal cycles also change. An interesting point of view about the consequences of such evolving behaviours is to measure their effects in dissimilarity and complementarity indexes, as shown in Graph 12.

One of the most expected findings is the high dissimilarity level for each of the seasonal cycles. Dissimilarity is increasing for weekly and monthly seasonal variations in such a way that the dissimilarity level at the end of the sample is near to its maximum level. However, there is not a clear trend in the dissimilarity level between the yearly seasonal cycles, although dissimilarity also increases towards the end of the sample period.

The high levels of dissimilarity between weekly and monthly seasonal cycles seem to be more related to differences in magnitude of seasonal variations, though the segments of the seasonal period with opposite-sign seasonal effects are not the cause of high levels of complementarity. A noticeable increase in the level of complementarity is observed only from the middle of 2018, and especially for the monthly seasonal cycles.

Higher complementarity levels are observed in the yearly cycles, although these levels decrease from 2015 to reach a minimum level in 2018 and increase again in 2019. As shown in Graph 6, according to the forecasts of yearly seasonal effects, the increase in complementarity level is related to the location of the periods with negative and positive seasonal effects in each series. In 2015, negative seasonal effects are observed in Germany from nearly the beginning of the year and almost until the beginning of the summer period, whereas in Spain this period is shorter because it begins later and finishes before. In 2019, positive seasonal effects disappear in Spain when the summer period finishes, whereas in Germany these positive seasonal effects remain beyond the summer period.

Graph 12: Dissimilarity and complementarity indexes for seasonal variations



4. Concluding remarks

The daily series of banknote net issues by the German and the Spanish national banks present salient features that are probably related to several issues that explain money demand in each case. As regards long-term behaviour, an upward trend is clearly observed in the German case, whereas an opposite trend is reducing net issues by the Spanish central bank up to the COVID pandemic. However, the contribution of this paper is focused on the detection of differences in seasonal patterns.

Although both series seem to be characterized by the presence of weekly, monthly and yearly seasonal variations, the relevance of each of these three seasonal patterns is always higher in the Spanish case. Furthermore, the evolution of these seasonal variations is also different in both cases. According to the extreme values of the seasonal effects, weekly seasonal variation has been dampening over time in Germany, whereas this seasonal variation seems to be increasing in Spain.

Similar behaviour is also observed regarding monthly and yearly seasonal patterns, although the evolution in the German case seems to have reversed over the last few years of the sample period. By assessing the areas measuring the magnitude of these seasonal variations, a clearer conclusion can be obtained about the increasing relevance of these seasonal variations in the Spanish case, whereas stability is a better description of the evolution observed in Germany. Of course, these differences in the magnitude of seasonal variations cause high levels of dissimilarity between each of the three seasonal patterns in both cases. In fact, the values of the dissimilarity index increase. However, the low values of complementarity indexes for weekly and monthly seasonal variations reveal that there are no noticeable differences in the shape of these seasonal patterns, although these differences seem to have been increasing from 2018. On the other hand, the higher values of the complementarity index between yearly seasonal patterns for both series at the beginning of the sample period are decreasing in the second half of this period; and the forecasted increase in 2019 reveals different changes in the shape of the yearly seasonal variation in Germany and Spain.

The economic reasons behind these different behaviours of seasonal patterns in net issues by German and Spanish national banks go beyond the aim of this paper. However, the restricted evolving spline models and the indexes to compare seasonal variations explained in the methodological section are shown to be useful tools to detect such differences in a much clearer way compared with the observation of the original daily series. An interesting future line of research would consist of applying these models to the daily series corresponding to different banknote denominations. In this sense, the application of dissimilarity and complementarity indexes would be very valuable to estimate the shares of transactional or non-transactional demand.

Declaration of AI use

The authors declare that AI tools have not been used in the preparation of this manuscript.

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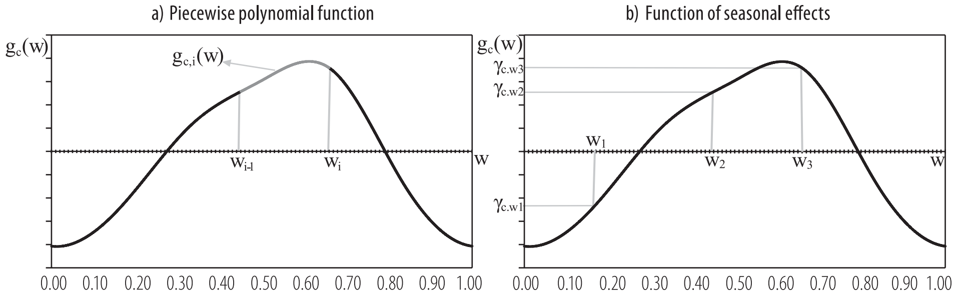
Appendix

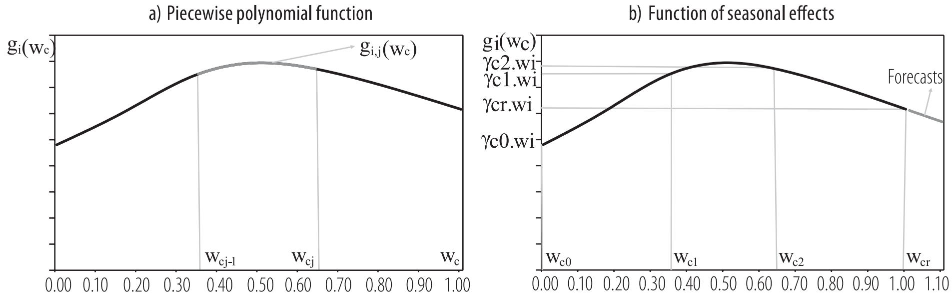
Restricted evolving spline model

Graph A.1: Seasonal effects over a whole seasonal cycle (periodic spline function)



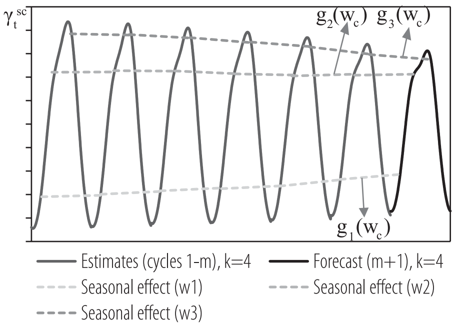
Note: In Figure A.1.a the spline is defined as a piecewise polynomial function in four segments (k=4). In Figure A.1.b the spline is defined as a function of the values of seasonal effects in three knots.

Graph A.2: Seasonal effects at a specific proportion of the seasonal cycle (non-periodic spline function)



Note: In Figure A.2.a the spline is defined as a non-periodic piecewise polynomial function in three segments (r=3). In Figure A.2.b the spline is defined as a function of the values of seasonal effects at proportion wi of the seasonal period in specific cycles. Forecasts for seasonal effects at the same proportion of the seasonal period in future seasonal cycles are obtained from this function.

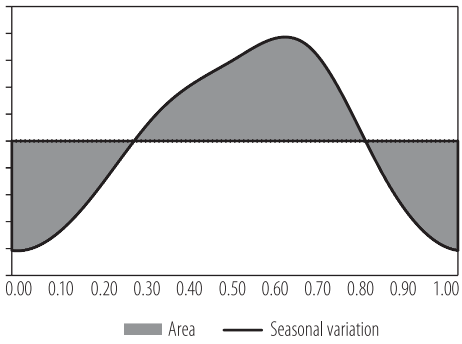
Graph A.3: Restricted evolving spline model and forecast



Note: An evolving seasonal pattern is obtained by assuming k=4 and r=3. Forecast of the whole seasonal pattern one seasonal cycle ahead is also obtained.

Dissimilarity and complementarity indexes

Graph A.4: Magnitude of the seasonal variation throughout the seasonal cycle c (A_c^{sc})



Graph A.5: Dissimilarity and complementarity between seasonal variations throughout seasonal period c ($DA_c^{sc,X,Y}$, $CA_c^{sc,X,Y}$)