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## **Denoised Inflation: A New Measure of Core Inflation<sup>1</sup>**

**Abstract:** Existing measures of core inflation ignore a part of ‘should be’ the core inflation. Exclusion based measures ‘exclude’ a part of persistent inflation inherently existing in the excluded part whereas filter based measures ‘filter-out’ the cyclical part also rather than the irregular component only. This study proposes a new idea to define and measure core inflation – noise free inflation or denoised inflation. As against considering only trend to define core inflation, this study proposes using cyclical component also to be part of core inflation. If core inflation is to be useful, for monetary policy making, as an indicator of underlying inflation, it has to include demand related component of inflation associated with current economic cycle. By using wavelet analysis approach to decompose seasonally adjusted price index into noise, cyclical component and trend, we estimate a denoised inflation series for Pakistan for the period July 1992 to June 2017. Since denoised inflation passes ‘statistical’ as well as ‘theoretical’ tests necessary for a series to be core inflation, we think it can be used as a new core inflation measure for Pakistan. This can also be estimated and tested for any country.

**Key Words:** Estimation of Cyclical Component, Inflation, Monetary Policy

**JEL Code:** C19, E31, E52.

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## 1. Introduction

During the last couple of decades, price stability has become the only objective of many central banks. For the central banks pursuing multiple objectives, it is has become the main objective. State Bank of Pakistan (SBP) is responsible to achieve government's targets of real GDP growth and inflation along with ensuring financial stability. SBP targets 12-month average of year on year (YoY) consumer price inflation in Pakistan.<sup>2</sup> Pakistan Bureau of Statistics (PBS) compiles the data of consumer price index (CPI) on monthly basis by collecting prices of 487 commodities from 76 Markets of 40 cities of Pakistan., Like many other macroeconomic time series, CPI is composed of underlying trend, cyclical component, seasonal fluctuation, and irregular movements. SBP's targeting of 12-month average clearly indicates that SBP cares for a sort of underlying/trend inflation. This indicator is less sensitive to transient changes in prices (compared to headline inflation). Reason is simple that monetary policy is forward looking and that one-off shock(s) in prices may have temporary effect on inflation that need not necessarily require any policy action as emphasized in Bank for International Settlements (1999)<sup>3</sup>. SBP uses Non-food Non-energy (NFNE) inflation, 20% trimmed inflation, and inflation in relatively stable component (RSC) of CPI as measures of core inflation in Pakistan.

Economics researchers, national statistical agencies and monetary policy makers use the concept of core inflation widely. The Bureau of Labour Statistics (USA) has been publishing CPI ex food and energy since 1978. The reporting of CPI ex-food has even longer history in developed countries. Regarding core inflation in Pakistan, the first published study was Tahir (2003).

When it comes to conceptual definition of core inflation it is yet to find a universally agreed upon definition, however. There exists vast literature on how to define and measure inflation that is free of one-off and/or temporary shocks. According to Eckstein (1981) core inflation ...[is]... *“the rate that would occur on the economy's long term growth path, provided the path were free of shocks, and on the state of demand were neutral in the sense that markets were in the long run equilibrium”*. In other words, core inflation is steady state concept and deviations from the steady state are either demand fluctuations or exogenous shocks (Par-

<sup>2</sup> Like South African Reserve Bank. In South Africa, the inflation target is specified as an average rate of increase in consumer prices.

<sup>3</sup> “Central banks should only resist persistent sources of inflationary pressures and not be concerned with short term and reversible movements in prices and the inflation rate.” Foreword (Bank for International Settlements, 1999).

kin, 1984). Thus, Eckstein defines core inflation as that part of inflation which can be associated with the long term result of monetary dynamics. It is separately defined from the demand related component of inflation – the one that is associated with the gap between actual and potential level of economic activity and is main concern of current monetary policy making – and the exogenous supply shock(s) related component of inflation. Eckstein suggests econometric approach to estimate the core inflation. So, by Eckstein definition, core inflation itself is an underlying steady state inflation rate; and not *an indicator* of underlying inflation trend as explained by Clark (2001).

If a measure of core inflation is to be useful for (short term) monetary policy making as an indicator of underlying inflation, it has to include the demand related component of inflation that is associated with the current economic heat/slack in the country. Short term monetary policy decision(s) will set the path of future steady state inflation rate and thus to have any measure of core inflation, we need to drop only that part of inflation which is the outcome of exogenous supply shocks and retain the demand related component.

There is another strand of literature related to the concept of core inflation - through persistent component of inflation. This part of inflation is the one that has no long-term impact on real output (Quah and Vahey, 1995) and can be measured through different ways. Generally ‘transient’ part is removed either through some exclusion (of commodities with generally volatile prices) approach or through smoothing using some statistical filtering. We believe both the approaches ignore a part of ‘should be’ the core inflation, in fact, exclusion based measures ‘exclude’ a part of the persistent inflation inherently existing in the excluded part; and filter based measures ‘filter-out’ the cyclical part also rather than subtracting the irregular component only. This can be checked by simply testing if the usually excluded component - food and energy inflation in the country - is persistent, i.e. not that temporary to exclude. In case food & energy inflation is persistent, which in fact is the case of Pakistan, our exclusion-based measure of core inflation will be ‘leaving out’ the persistent part of inflation in an attempt to estimate the core inflation that conceptually should be the ‘overall’ persistent component of inflation in the country. In such attempt, we may exclude an information permanently which contains not only noise but signal as well.

Moreover, exogenous supply shocks are not always related to any specific part of the CPI basket, like food and energy. Exclusion based measures of core inflation ‘retain’ the outcome of exogenous supply shocks in the retained part of the CPI basket, like in NFNE basket. If underlying trend in overall inflation is being set by food and energy inflation, as had been the case for some countries (Cecchetti

and Moessner, 2008), exclusion-based measures of inflation excludes a part of the 'should be' core inflation. When food & energy price changes become persistent (rather than temporary) then they are very likely to impact inflation over the horizon relevant to monetary authorities (Cecchetti and Moessner). Food & energy inflation also affect the inflation expectations, as we have observed in the case of Pakistan<sup>4</sup>, and thus creates second round effects and becomes important for monetary authorities to pay attention.

Thus, we think there is need for clearly defining what is meant by core inflation and how it should be measured. This is even more important when core inflation is measured using some statistical filtering. In statistical filtering, not only the irregular part is removed from a seasonally adjusted prices series but the cyclical component is also dropped and only the trend is considered as a one and used as core inflation. There is no question on seasonal adjustment. However, we think, with reference to use as an estimate of core inflation, only the irregular component merits to be excluded as a temporary part and not the cyclical component – cycles are not that temporary. From the literature on booms and busts in the US economy, we know that in 33 business cycles in the USA during 1854-2009, expansion and contraction periods averaged 38.7 and 17.5 months, respectively<sup>5</sup>. If it the case for income, similar duration periods should hold for prices and there is some evidence in the literature in this context which has been documented by Cashin et al. (2002) - slumps (in global commodity prices) last longer than booms. Therefore, we define core inflation as a 'denoised inflation measured from a seasonally adjusted consumer price index'<sup>6</sup>. In addition to redefining the theoretical concept of what is in the core inflation, we also contribute to the literature by implementing a rarely used approach to measure core inflation – wavelet analysis approach (WAN). But, we use this approach based upon our own idea to estimate the non-temporary part – cycle and trend (CAT) constitute the core inflation – in the case of Pakistan using monthly consumer price index from July 1992 to June 2017. As against other statistical approaches to decompose a time series, WAN does not require strong assumptions pertaining to noise or the trend in a time series and thus has no problem in analyzing a time series with

<sup>4</sup> Results are in Table 11 of the Appendix III.

<sup>5</sup> <http://www.nber.org/cycles/>

<sup>6</sup> We think our idea of defining what is 'core inflation' may have vast implications for macroeconomics literature. For example, there may be a need to rethink to proxy permanent income in the context of Permanent Income Hypothesis. 'Cycle is the trend' concept (Aguilar and Gopinath, 2007) lends significant support to our suggestion to exclude 'seasonal and irregular' components only and proxy the 'permanent' income by joint evolution of 'cyclical and trend' components. To test our argument one may explore the behaviour of consumption during various stages of business cycles and see consumption in not immune from cyclical in income.

non-stationarity, regime shifts and isolated shocks (Dowd et al. 2011). Moreover, unlike other conventional filters, including Fully Modified HP filter of Iqbal & Hanif (2017), WAN does not contaminate cyclical component with noise as it separately estimates the noise part.

There could have been different names from WAN inflation to CAT inflation, but we preferred to call it 'denoised' inflation to emphasize the fact that only the 'noise' or 'irregular' or 'transient' or 'one-off component' is excluded. We test the estimated denoised inflation measured for Pakistan to usual statistical diagnostic procedure for it to qualify to be core inflation, as described in (Marques *et al.* 2003), and found it passing all such tests. Three exclusion based core inflation measures which are currently being reported in Pakistan – NFNE, Trimmed, and 'Relatively Stable Components of CPI' (RSC) measures – are found unsuccessful in this statistical evaluation process. Moreover, we also investigated if the denoised inflation is related to monetary policy stance in Pakistan<sup>7</sup>. We found it 'linked' compared to absence of any such link of any of the three exclusion based core inflation measures for the case of Pakistan.

After the introduction, this study is organised as follows. Section 2 discusses data used in this study. In section 3, we explain the methodology for the estimation of denoised inflation. In section 4, estimated denoised inflation is presented. Section 5 contains the results of evaluation of denoised inflation and existing measures of core inflation by statistical as well as theoretical standards. Section 6 concludes the paper.

## 2. Data

We used the data of CPI for Pakistan from July 1992 to June 2017 that is compiled by PBS to estimate core inflation. For the purpose of comparison of our estimated core inflation with the currently used measures of core inflation, we used NFNE, 20% trimmed mean and RSC core inflation series from SBP<sup>8</sup>. The data for last two measures is available only from July 2008 to June 2017. In order to make 'theoretical' evaluation of the new and existing measures of core inflation, we also used data of call money rate (CMR) and broad money supply (M2), which is collected from Statistical Bulletin of SBP.

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<sup>7</sup> In comparison to the statistical evaluation of Marques et al. 2003, we call this exploration of core inflation measure and monetary policy stance nexus as 'theoretical evaluation' of core inflation measures.

<sup>8</sup> The data for last two measures is available only for Jul 2008 to June 2017.

### 3. Methodology

While revisiting Eckstein (1981) idea to exclude both the demand as well as supply related fluctuations in price increases to come up with core inflation, we propose to exclude only the exogenous supply shock(s) related component of inflation to estimate core inflation (as an indicator of underlying inflation). This cannot be estimated using an exclusion based measure of inflation as it will either ‘exclude’ some part of ‘persistent inflation or ‘retain’ exogenous supply shock related inflation in the retained basket. To measure core inflation in conformity with our idea, we proposed to estimate ‘denoised’ inflation from a seasonally adjusted CPI. To extract noise we decomposed the seasonally adjusted CPI by the using wavelet analysis.

Wavelets are mathematical expansions that transform data from the time domain into different layers of frequency levels. Conventionally, economists consider only two scales in a time series: the short run and the long run. There are actually more time scales in between the short run and the long run horizon of a time series (Dalkir, 2004). Through wavelet decomposition analysis, we can split a time series into different frequency zones, very high frequency (noise part) and very low frequency (smooth part) and moderate frequency zone (cyclical part). [Multiresolution] wavelet analysis is a useful tool to decompose an economic time series into trend, cycle, and noise (Yogo, 2008). A seasonally adjusted time series  $P_t$  can be decomposed as  $P_t = S_t^J + \sum_{j=1}^J D_t^j$ . Where  $S_t^J$  is actually secular trend being with periodicity greater than  $2^{J+1}$  and  $D_t^j$  denotes cycles with periodicity between  $2^j$  and  $2^{j+1}$ . We take  $j \leq J$  and that  $J = 6, 4$  and  $2$  for monthly, quarterly and annual data series respectively [Yogo (2008), Crowley (2010) and Iqbal and Hanif (2017)]. Since we are working with monthly time series, we set  $J = 6$ . For monthly data,  $S_t^J$  is the trend component (with periodicity greater than  $2^{6+1}$  or 128);  $D_t^4$ ,  $D_t^5$  and  $D_t^6$  are the free of noise cycle components with periodicity of 16-32, 32-64 and 64-128 months respectively and  $D_t^1$ ,  $D_t^2$  and  $D_t^3$  denote the high frequency noise component with periodicity less than 16 months. Hence for deseasonalized monthly data series

$$P_t = S_t^6 + \sum_{j=4}^6 D_t^j + \sum_{j=1}^3 D_t^j = \text{trend} + \text{cycle} + \text{noise}$$

There are different wavelet types for different applications including Haar wavelet and Daubechies wavelet (Daubechies, 1992, Misiti *et al.* 2000). To apply wavelet analysis, we first select a suitable wavelet type. The Haar wavelet is the simplest one but it is not orthogonal, whereas Daubechies wavelet is orthogonal. In this study, we use Daubechies’s Daub4 wavelet which is orthogonal symmetric wave-

let filter<sup>9</sup>. To estimate all the three components of deseasonalized CPI using the WAN, we use the MATLAB code provided by Iqbal & Hanif (2017).

### 4. Estimated Denoised Inflation

We decompose the deseasonalized CPI for Pakistan from July 1992 to June 2017 using aforementioned wavelet analysis approach. Year-on-year (YoY) change in CPI, its estimated trend, (noise free) cyclical and noise components are plotted for Pakistan in figures 1a to 1d. We estimate the denoised consumer prices index by removing the noise from seasonal adjusted CPI.

Figure 1a: Headline inflation



Figure 1b: Trend Component of head line inflation

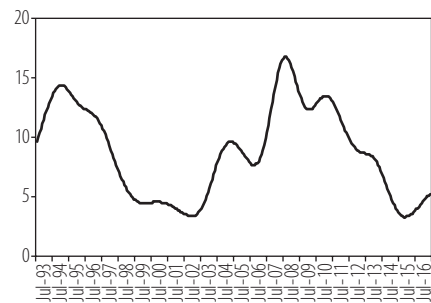


Figure 1c: Noise free cyclical component of head line inflation

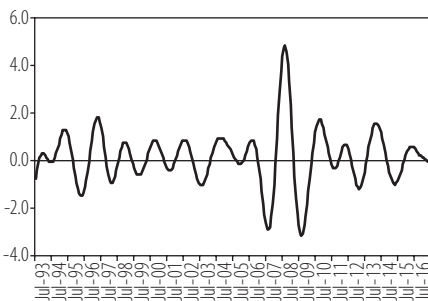
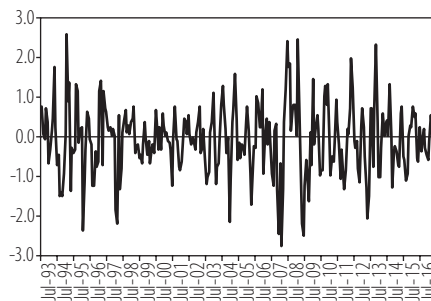


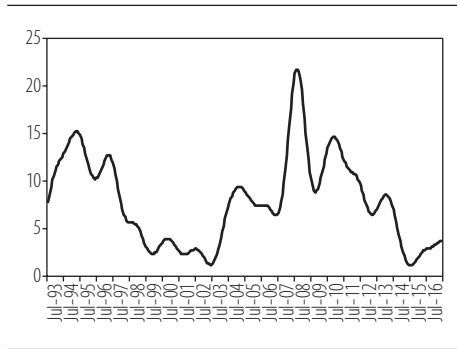
Figure 1d: Noise part in head line inflation



<sup>9</sup> Technical details are available in Iqbal and Hanif (2017).

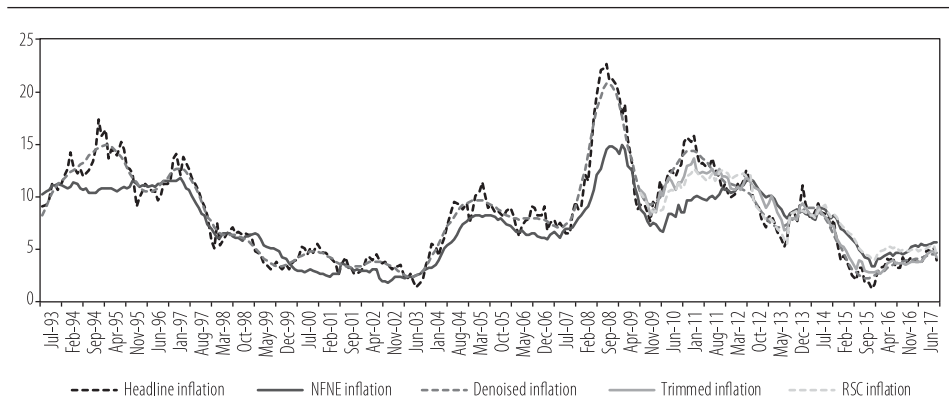
In Figure 2 we present YoY change in this denoised consumer prices index which we call denoised inflation.

**Figure 2: Denoised core inflation**



In Figure 3, we also show denoised inflation along with headline inflation and the three generally used core inflation series – NFNE inflation, 20% Trimmed mean inflation, and RSC-CPI inflation<sup>10</sup>. Time series data on denoised inflation in Pakistan is provided in the Appendix II for the period July 1992 to June 2017.

**Figure 3: Headline and Core inflation in Pakistan**



## 5. Evaluation of Denoised Inflation as a Measure of Core Inflation for Pakistan

In the following two subsections, we discuss statistical and theoretical evaluation of our proposed measure, with those which are currently used inflation rate series as core inflation in Pakistan.

<sup>10</sup> For details about currently used inflation series as core inflation in Pakistan see Tahir (2003) and Riazuddin *et al.* (2013).



## 5.1 Statistical Evaluation of Core Inflation Measures

Various studies propose different qualitative and quantitative characteristics for a measure to be used as core inflation. These include timely availability, credibility, verifiability, and understandability (Roger, 1998); real time computability and forward looking nature (Wynne, 1999); same mean of as that of headline inflation (Clark, 2001); ability to track movements in some sort of trend headline inflation [Cecchetti (1997), Bakhshi and Yates (1999), Vega and Wynne (2001)]; cointegration with headline inflation (Freeman, 1998); and headline inflation forecast-ability [Freeman (1998), Cogley (2000), Vega and Wynne (2001)].

When considering some of these characteristics as pre-requisite and criticizing some as unjustifiable, Marques *et al.* (2003) proposed a formal set of testable conditions for a measure to serve as core inflation. These conditions are: i) the core and headline inflation series ( $\pi_t$ ) should be co-integrated with unitary coefficient; ii) the core inflation ( $\pi_t^*$ ) should be an attractor of headline inflation; and iii) the headline inflation should not be the attractor of the core inflation. Three conditions are described in the appendix I in algebraic manner. Moreover, recently Dowd *et al.* (2011) suggest adding a couple of more characteristics which are a) fewer turning points in core inflation (than the headline inflation series) and b) frequent crossing of headline inflation series.

We discuss the results for each of these statistical tests pertaining to various core inflation measures for Pakistan, including the newly proposed denoisied inflation.

## 5.2 Cointegration between Core and Headline Inflation

To test condition 1 of Marques *et al.* (2003), we proceed as follows. First, using ADF test, we established that the headline inflation and all the measure of core inflation under scrutiny are integrated of order 1 (see Table 2 in the appendix III). The ADF test results are reported in Table 2 which indicate that  $z_t = \pi_t - \pi_t^*$  is stationary for all the four measures of core inflation and hence all the core inflation series are co-integrated with headline inflation.

The next step of condition 1 is to test that  $E(z_t) = 0$ . The second column of Table 3 (in the appendix III) shows the mean of  $z_t$  and its standard error are in parenthesis. Table 3 (in the appendix III) results indicates that  $z_t$  for trimmed mean core measure and denoisied inflation is zero while for NFNE and RSC-CPI inflation it is significantly different from zero. Hence NFNE and RSC-CPI core inflation do not fulfil the first condition of Marques *et al.* (2003) i.e. Although these are

co-integrated with headline inflation but do not fulfil the (partial) condition that  $E(z_t) = 0$ , while other two core inflation measure, denoised and trimmed core inflation, passed the condition 1 of core inflation evaluation.

### 5.3 Core Inflation is an Attractor of Headline Inflation

In order to test this condition that if core inflation measure is an attractor of headline inflation, following ECM equation is estimated for each of the three inflation series separately.

$$\Delta\pi_t = \alpha + \sum_{i=1}^m \beta_i \Delta\pi_{t-i} + \sum_{j=1}^n \gamma_j \Delta\pi_{t-j}^* - \varphi(\pi_{t-1} - \pi_{t-1}^*) + \varepsilon_t$$

The null hypothesis to be tested is that  $\varphi = 0$  and rejection of null hypothesis implies that core inflation measures are the attractor of headline inflation. From results presented in Table 4 (in the appendix III), it is evident that  $\varphi = 0$  for NFNE core inflation, trimmed and RSC-CPI core inflation. However, we reject the hypothesis that  $\varphi = 0$  in case of denoised inflation. This implies that denoised inflation is an attractor of headline inflation and headline inflation does converge to estimated denoised inflation in the long run. NFNE and RSC-CPI measures of core inflation do not satisfy this condition. Trimmed inflation, which passed the first condition, failed in fulfilling the condition 2.

### 5.4 Headline Inflation Does not Attract Core Inflation

The third condition is to ensure that the estimated core inflation does not converge to headline inflation. To test this condition, we estimated the following ECM for each of the core inflation measures.

$$\Delta\pi_t^* = \alpha + \sum_{i=1}^m \beta_i \Delta\pi_{t-i} + \sum_{j=1}^n \gamma_j \Delta\pi_{t-j}^* - \lambda(\pi_{t-1} - \pi_{t-1}^*) + \varepsilon_t$$

The hypothesis, to check this condition, is that  $\lambda = 0$ . Results in Table 5 (in the appendix III) indicate that we are unable to reject the null  $\lambda = 0$  only for denoised inflation. NFNE, Trimmed core inflation and RSC are unable to pass condition 3.

The next step of this condition is to ensure the strong exogeneity of core inflation measure. In order to test strong exogeneity, we have to check that  $\pi_t$  does not Granger cause  $\pi_t^*$ . We apply Granger non-causality test for headline and denoised inflation pair and test for strong exogeneity condition. We can see from

Table 6 (in the appendix III) that denosed inflation causes the headline inflation while headline inflation does not cause denosed inflation and that the denosed measure of core inflation satisfies the strong exogeneity condition.

These findings suggest that the denosed inflation passes all the conditions of core inflation which are taken from Marques et al. (2003) whereas none of the currently published core inflation series passes these tests. In addition to the conditions discussed by Marques et al., denosed inflation also satisfies a couple of more conditions mentioned in Dowd et al. (2011) as shown in Table 7 (in the appendix III).

## 5.5 Theoretical Evaluation of Core Inflation Measures

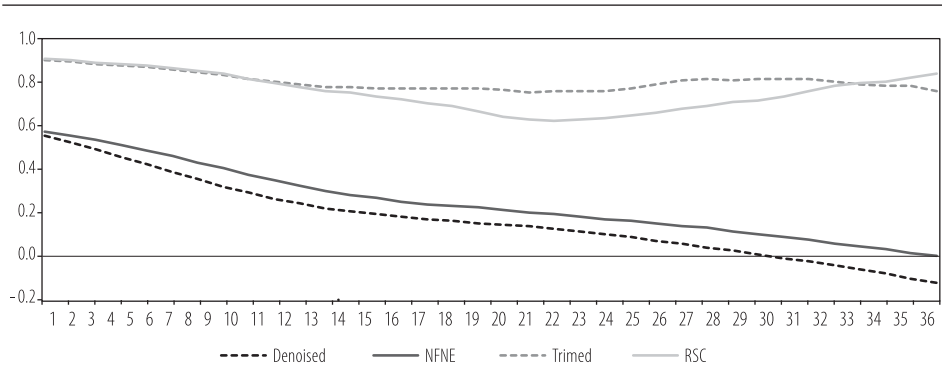
Unlike the economic concept associated with measurement of inflation – change in the cost of living – there is no substantial theoretic/conceptual framework linked with the measurement of core inflation. However, some researchers have linked core inflation with quantity theory of money in the sense that changes in stock of money should presumably affect prices' dynamics equiproportionately (Wynne, 1999). Since there are lags in the impact of monetary policy upon real side of the economy, we can assume that such equiproportionate response of price is free from one off components. Thus, the equiproportionate impact of monetary growth should be reflected in the core inflation. In this study, we used this concept to make a 'monetary' evaluation of core inflation indicators for Pakistan, including the one we have suggested. We have implemented this in two stages.

First, we looked at the dynamic correlation coefficients of short term interest rate with each of core inflation measure evaluated in this study to see which one is best (negatively) correlated with the short term interest rate. In this approach, we observed that it is the denosed inflation that has negative correlations with short term interest rate (call money rate) after a certain lag of time; compared to all the other three measures of core inflation under evaluation in this study (figure 4).

Second, we have applied bivariate Granger non-causality test to see if core inflation is really caused by the change in broad money supply growth; and not the other way round (as argued by Bryan and Cecchetti (1994)). We find that monetary growth in Pakistan causes denosed inflation in the country without showing reverse causality (Table 8, Appendix III). Whereas none of the other three measures of core inflation is found to be caused by broad money growth in the country. It can be easily seen from the Table 9 that denosed inflation is better correlated with the broad money growth rate as compared to trimmed and RSC measures of core inflation. On the face of it, NFNE inflation indicator is more

correlated with the broad money growth rate compared to denoised inflation. However, the observed higher correlation of NFNE inflation is without having any sensible causal relationship with money supply growth as already shown in Table 8. Moreover, when we compare the correlation coefficients of broad money growth rate with excluded part inflation of both of different measures (Table 10, Appendix III), we find that it is only the denoised inflation which minus uncorrelated (with the money supply) part as Food & energy inflation is significantly correlated with money supply growth at different lags.

**Figure 4: Correlation coefficients between call money rate and different measures of core inflation**



## 6. Conclusion

Main responsibility of monetary authority in an economy is price stability. Many central banks are following inflation targeting regimes for this purpose. It is less desirable for a central bank to revise its monetary stance to address such temporary changes in inflation that result from exogenous supply shocks. As a measure of changes in prices that is free of non-monetary shocks related inflation, many central banks monitor core inflation. This measure serves as an indicator of underlying inflation (rather than the underlying trend in inflation). Various existing measures of core inflation suffer from either excluding a part of prices' basket (without noticing if something more than the impact of exogenous supply shocks is being excluded) or from removing cyclical component along with the noisy part from the headline inflation (without noticing that demand related part is also getting filtered out along with supply shock related component). Hence, it is important that central banks measure accurately what they actually intend to monitor as core inflation. Otherwise central bank may consider the under-/over-

estimated (core inflation) measure as to which headline inflation will converge and thus may take unwarranted action(s).

In this study, a new approach to estimate core inflation has been proposed. We use wavelet analysis to decompose deseasonalized consumer price index into different time scale zones and obtain a denoised index to calculate denoised inflation. This measure is conceptually relevant in the sense that it filters out exogenous supply shocks related component of headline inflation only, and not the response of prices to changing demand levels in the country. Statistically, neither it requires excluding a specific composition of prices' basket permanently at micro level nor it forces to filter out cyclical signals from the general prices at macro level.

We have estimated denoised inflation for Pakistan for the period of July 1992 to June 2017. When evaluating our proposed denoised inflation and the currently reported core inflation indicators for Pakistan, we find that only the denoised inflation series passes the generally considered statistical and theoretical tests of core inflation for the case of Pakistan. Denoised inflation is actually what central banks want to monitor as core inflation and not the NFNE or Trimmed or RSC-CPI inflation.

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## Appendix I

### Evaluation Criteria Adopted in this Study (Marques et al. 2003)

If, at any given time  $t$ , the headline inflation rate,  $\pi_t$ , is composed of two components; a permanent component ( $\pi_t^*$ ), named core inflation, and a temporary/transitory component  $v_t$  i.e.  $\pi_t = \pi_t^* + v_t$ . The first condition is that the two series are co-integrated with unit co-efficient i.e.  $z_t = \pi_t - \pi_t^*$  is stationary with zero mean.

The second requirement is that the core inflation is an attractor of headline inflation i.e. head line inflation must converge to core inflation in the long run. To test this condition, Marques *et al.* (2003) proposed to estimate the following error-correction model:

$$\Delta\pi_t = \alpha + \sum_{i=1}^m \beta_i \Delta\pi_{t-i} + \sum_{j=1}^n \gamma_j \Delta\pi_{t-j}^* - \varphi(\pi_{t-1} - \pi_{t-1}^*) + \varepsilon_t$$

and test the hypothesis that  $\varphi = 0$ . The rejection of hypothesis suggests that core inflation ( $\pi_t^*$ ) is an attractor of headline inflation ( $\pi_t$ ) and headline inflation does converge to estimated core inflation in the long run.

The third requirement is the strong exogeneity of  $\pi_t^*$ . It implies that the error correction term in ECM does not appear in the equation for  $\pi_t^*$  and also that  $\pi_t$  does not Granger cause  $\pi_t^*$ . In other words, in the following model

$$\Delta\pi_t^* = \alpha + \sum_{i=1}^m \beta_i \Delta\pi_{t-i} + \sum_{j=1}^n \gamma_j \Delta\pi_{t-j}^* - \lambda(\pi_{t-1} - \pi_{t-1}^*) + \varepsilon_t$$

We must have  $\lambda = 0$ . Failure to reject this hypothesis suggests that  $\pi_t^*$  is weakly exogenous.



## Appendix II

**Table 1: Denoised Inflation Series for Pakistan (July 1992 - June 2017)**

Jul-93	8.13	Jul-01	3.48	Jul-09	10.11
Aug-93	8.88	Aug-01	3.36	Aug-09	9.52
Sep-93	9.56	Sep-01	3.29	Sep-09	9.17
Oct-93	10.13	Oct-01	3.26	Oct-09	9.04
Nov-93	10.61	Nov-01	3.28	Nov-09	9.13
Dec-93	11.00	Dec-01	3.33	Dec-09	9.39
Jan-94	11.34	Jan-02	3.41	Jan-10	9.79
Feb-94	11.62	Feb-02	3.50	Feb-10	10.30
Mar-94	11.87	Mar-02	3.61	Mar-10	10.87
Apr-94	12.10	Apr-02	3.70	Apr-10	11.47
May-94	12.31	May-02	3.77	May-10	12.06
Jun-94	12.53	Jun-02	3.80	Jun-10	12.61
Jul-94	12.77	Jul-02	3.80	Jul-10	13.10
Aug-94	13.02	Aug-02	3.75	Aug-10	13.50
Sep-94	13.29	Sep-02	3.65	Sep-10	13.81
Oct-94	13.59	Oct-02	3.51	Oct-10	14.01
Nov-94	13.88	Nov-02	3.34	Nov-10	14.09
Dec-94	14.14	Dec-02	3.14	Dec-10	14.06
Jan-95	14.37	Jan-03	2.93	Jan-11	13.93
Feb-95	14.54	Feb-03	2.73	Feb-11	13.70
Mar-95	14.61	Mar-03	2.56	Mar-11	13.41
Apr-95	14.59	Apr-03	2.43	Apr-11	13.06
May-95	14.46	May-03	2.36	May-11	12.69
Jun-95	14.23	Jun-03	2.37	Jun-11	12.32
Jul-95	13.90	Jul-03	2.47	Jul-11	11.97
Aug-95	13.50	Aug-03	2.66	Aug-11	11.66
Sep-95	13.04	Sep-03	2.94	Sep-11	11.40
Oct-95	12.54	Oct-03	3.31	Oct-11	11.20
Nov-95	12.04	Nov-03	3.76	Nov-11	11.05
Dec-95	11.57	Dec-03	4.26	Dec-11	10.94
Jan-96	11.14	Jan-04	4.81	Jan-12	10.84
Feb-96	10.78	Feb-04	5.39	Feb-12	10.75
Mar-96	10.50	Mar-04	5.97	Mar-12	10.64
Apr-96	10.32	Apr-04	6.53	Apr-12	10.48
May-96	10.25	May-04	7.07	May-12	10.26

Jun-96	10.29	Jun-04	7.56	Jun-12	9.98
Jul-96	10.42	Jul-04	8.01	Jul-12	9.63
Aug-96	10.64	Aug-04	8.39	Aug-12	9.23
Sep-96	10.94	Sep-04	8.72	Sep-12	8.80
Oct-96	11.27	Oct-04	8.99	Oct-12	8.36
Nov-96	11.62	Nov-04	9.20	Nov-12	7.94
Dec-96	11.94	Dec-04	9.35	Dec-12	7.57
Jan-97	12.21	Jan-05	9.45	Jan-13	7.27
Feb-97	12.38	Feb-05	9.50	Feb-13	7.05
Mar-97	12.43	Mar-05	9.50	Mar-13	6.95
Apr-97	12.35	Apr-05	9.45	Apr-13	6.95
May-97	12.12	May-05	9.35	May-13	7.04
Jun-97	11.75	Jun-05	9.21	Jun-13	7.22
Jul-97	11.25	Jul-05	9.05	Jul-13	7.47
Aug-97	10.65	Aug-05	8.85	Aug-13	7.75
Sep-97	9.99	Sep-05	8.64	Sep-13	8.04
Oct-97	9.30	Oct-05	8.43	Oct-13	8.33
Nov-97	8.62	Nov-05	8.23	Nov-13	8.56
Dec-97	8.00	Dec-05	8.06	Dec-13	8.74
Jan-98	7.46	Jan-06	7.91	Jan-14	8.83
Feb-98	7.02	Feb-06	7.81	Feb-14	8.83
Mar-98	6.69	Mar-06	7.75	Mar-14	8.72
Apr-98	6.46	Apr-06	7.72	Apr-14	8.52
May-98	6.33	May-06	7.74	May-14	8.22
Jun-98	6.26	Jun-06	7.76	Jun-14	7.84
Jul-98	6.24	Jul-06	7.80	Jul-14	7.38
Aug-98	6.23	Aug-06	7.83	Aug-14	6.86
Sep-98	6.22	Sep-06	7.83	Sep-14	6.30
Oct-98	6.17	Oct-06	7.80	Oct-14	5.71
Nov-98	6.06	Nov-06	7.73	Nov-14	5.12
Dec-98	5.90	Dec-06	7.62	Dec-14	4.54
Jan-99	5.67	Jan-07	7.47	Jan-15	4.00
Feb-99	5.40	Feb-07	7.30	Feb-15	3.51
Mar-99	5.09	Mar-07	7.14	Mar-15	3.09
Apr-99	4.75	Apr-07	7.01	Apr-15	2.75
May-99	4.42	May-07	6.95	May-15	2.49
Jun-99	4.11	Jun-07	7.00	Jun-15	2.33

Jul-99	3.84	Jul-07	7.19	Jul-15	2.24
Aug-99	3.62	Aug-07	7.55	Aug-15	2.24
Sep-99	3.45	Sep-07	8.12	Sep-15	2.32
Oct-99	3.36	Oct-07	8.90	Oct-15	2.44
Nov-99	3.33	Nov-07	9.89	Nov-15	2.61
Dec-99	3.37	Dec-07	11.07	Dec-15	2.80
Jan-00	3.46	Jan-08	12.41	Jan-16	2.99
Feb-00	3.60	Feb-08	13.85	Feb-16	3.18
Mar-00	3.78	Mar-08	15.32	Mar-16	3.35
Apr-00	3.97	Apr-08	16.75	Apr-16	3.50
May-00	4.17	May-08	18.03	May-16	3.61
Jun-00	4.36	Jun-08	19.10	Jun-16	3.71
Jul-00	4.52	Jul-08	19.88	Jul-16	3.78
Aug-00	4.65	Aug-08	20.31	Aug-16	3.83
Sep-00	4.73	Sep-08	20.37	Sep-16	3.89
Oct-00	4.77	Oct-08	20.05	Oct-16	3.94
Nov-00	4.74	Nov-08	19.38	Nov-16	4.00
Dec-00	4.67	Dec-08	18.41	Dec-16	4.06
Jan-01	4.55	Jan-09	17.22	Jan-17	4.15
Feb-01	4.39	Feb-09	15.90	Feb-17	4.23
Mar-01	4.20	Mar-09	14.53	Mar-17	4.33
Apr-01	4.01	Apr-09	13.20	Apr-17	4.42
May-01	3.81	May-09	11.98	May-17	4.50
Jun-01	3.63	Jun-09	10.94	Jun-17	4.57

## Appendix III: Results of Evaluation of Estimated Core Inflation Series for Pakistan.

Table 2: Unit Root Test for Headline and Core Inflation

	Variable	t-Statistic	P-value	Remarks
At Level	$\pi_t$	-1.16	0.24	Non-Stationary
	$\pi_t^{*N}$	-1.09	0.24	Non-Stationary
	$\pi_t^{*T}$	-1.98	0.60	Non-Stationary
	$\pi_t^{*R}$	-1.91	0.64	Non-Stationary
	$\pi_t^{*D}$	-1.59	0.11	Non-Stationary
1 <sup>st</sup> Difference	$\pi_t$	-8.96	0.00	Stationary
	$\pi_t^{*N}$	-7.39	0.00	Stationary
	$\pi_t^{*T}$	-8.89	0.00	Stationary
	$\pi_t^{*R}$	-8.40	0.00	Stationary
	$\pi_t^{*D}$	-4.29	0.00	Stationary

Table3: Zero Mean Stationarity Test of  $z_t = \pi_t - \pi_t^*$

	Mean (S.E.)	ADF Statistics	P-value (ADF test)	Remarks
$z_t = \pi_t - \pi_t^{*N}$	0.88(0.13) <sup>a</sup>	-2.04	0.04	Stationary
$z_t = \pi_t - \pi_t^{*T}$	-0.09(0.11)	-2.90	0.00	Stationary
$z_t = \pi_t - \pi_t^{*R}$	-0.34(0.16)	-3.41	0.04	Stationary
$z_t = \pi_t - \pi_t^{*D}$	0.02(0.05)	-8.69	0.00	Stationary

<sup>a</sup> Standard error of mean is in parenthesis.

**Table 4: Results of ECM – Dependent Variable is  $\Delta\pi_t$**

	$\pi_t^{*N}$		$\pi_t^{*T}$		$\pi_t^{*R}$		$\pi_t^{*D}$	
	coefficient	t-statistics	coefficient	t-statistics	coefficient	t-statistics	coefficient	t-statistics
C	0.02	0.39	-0.12	-1.55	-0.11	-1.47	0.04	1.46
$\Delta\pi_{t-1}$	0.07	1.01	0.23	1.26	0.30	2.36	4.95	8.68
$\Delta\pi_{t-2}$	0.03	0.51	0.02	0.10	-0.34	-2.56	4.54	8.43
$\Delta\pi_{t-3}$	0.07	1.12	0.58	3.24	0.15	1.09	3.73	7.40
$\Delta\pi_{t-4}$	0.03	0.45	0.14	0.75	-0.20	-1.51	3.12	6.78
$\Delta\pi_{t-5}$	-0.10	-1.58	0.19	1.00	-0.01	-0.04	2.47	6.05
$\Delta\pi_{t-6}$	0.02	0.35	-0.28	-1.61	-0.08	-0.68	1.71	4.91
$\Delta\pi_{t-7}$	-0.04	-0.66	-0.01	-0.09	0.06	0.48	1.02	3.58
$\Delta\pi_{t-8}$	0.09	1.58	0.23	2.42	0.29	2.61	0.74	3.19
$\Delta\pi_{t-9}$	0.01	0.18	-0.12	-1.28	-0.11	-1.05	0.38	2.17
$\Delta\pi_{t-10}$	0.01	0.12	0.20	2.25	0.28	2.48	0.05	0.37
$\Delta\pi_{t-11}$	0.03	0.44	0.00	-0.03	-0.03	-0.27	0.17	1.80
$\Delta\pi_{t-12}$	-0.57	-9.71	-0.42	-4.52	-0.42	-4.22	-0.23	-3.35
$\Delta\pi_{t-13}$	0.03	0.45	---	---	---	---	-0.18	-3.47
$\Delta\pi_{t-1}^*$	0.40	2.31	0.13	0.50	0.06	0.31	45.22	5.17
$\Delta\pi_{t-2}^*$	0.04	0.23	-0.62	-2.47	0.14	0.72	-137.29	-5.69
$\Delta\pi_{t-3}^*$	0.44	2.45	-0.57	-2.19	0.34	1.79	148.78	4.71
$\Delta\pi_{t-4}^*$	-0.25	-1.37	-0.20	-0.75	0.24	1.28	-120.44	-3.64
$\Delta\pi_{t-5}^*$	0.07	0.37	-0.08	-0.33	-0.12	-0.67	55.73	1.69
$\Delta\pi_{t-6}^*$	-0.01	-0.03	0.53	2.12	0.02	0.13	-50.74	-1.58
$\Delta\pi_{t-7}^*$	0.10	0.57	---	---	-0.18	-1.03	75.33	2.92
$\Delta\pi_{t-8}^*$	-0.10	-0.54	---	---	0.03	0.19	-38.39	-3.82
$\Delta\pi_{t-9}^*$	-0.10	-0.53	---	---	-0.02	-0.10	---	---
$\Delta\pi_{t-10}^*$	-0.06	-0.33	---	---	-0.40	-2.34	---	---
$\Delta\pi_{t-11}^*$	-0.10	-0.59	---	---	---	---	---	---
$\Delta\pi_{t-12}^*$	0.43	2.58	---	---	---	---	---	---
$(\pi - \pi^*)_{t-1}$	-0.05	-1.33	-0.07	-0.46	-0.02	-0.24	<b>-6.07</b>	<b>-10.17</b>

Table 5: Results of ECM – Dependent Variable is  $\Delta\pi_t^*$ 

	$\pi_t^N$		$\pi_t^T$		$\pi_t^R$		$\pi_t^D$	
	coefficient	t-statistics	coefficient	t-statistics	coefficient	t-statistics	coefficient	t-statistics
C	-0.03	-1.43	-0.09	-1.81	-0.03	-0.57	0.00	-0.48
$\Delta\pi_{t-1}$	0.04	1.80	0.06	0.49	0.30	3.46	0.00	-0.63
$\Delta\pi_{t-2}$	0.05	2.45	0.07	0.65	-0.06	-0.60	0.00	-0.70
$\Delta\pi_{t-3}$	0.02	1.03	0.12	1.46	0.01	0.15	0.00	-0.52
$\Delta\pi_{t-4}$	0.00	0.08	-0.02	-0.24	-0.17	-1.92	0.00	-0.38
$\Delta\pi_{t-5}$	-0.04	-2.01	0.04	0.58	-0.09	-1.18	0.00	-0.89
$\Delta\pi_{t-6}$	0.01	0.36	-0.09	-1.38	0.11	1.40	0.00	-0.95
$\Delta\pi_{t-7}$	-0.04	-1.89	-0.07	-1.15	0.02	0.20	0.00	-0.46
$\Delta\pi_{t-8}$	-0.01	-0.51	0.09	1.73	0.07	0.97	0.00	-0.77
$\Delta\pi_{t-9}$	-0.02	-0.98	-0.01	-0.10	-0.16	-2.26	0.00	-1.40
$\Delta\pi_{t-10}$	-0.03	-1.31	0.04	0.71	0.02	0.25	0.00	-0.99
$\Delta\pi_{t-11}$	-0.04	-2.18	0.04	0.65	---	---	0.00	-0.84
$\Delta\pi_{t-12}$	-0.05	-2.54	-0.29	-5.10	---	---	0.00	-2.97
$\Delta\pi_{t-13}$	0.00	-0.08	---	---	---	---	---	---
$\Delta\pi_{t-1}^*$	0.21	3.36	0.06	0.32	-0.16	-1.13	2.69	42.98
$\Delta\pi_{t-2}^*$	-0.04	-0.69	-0.34	-2.12	0.10	0.73	-1.87	-10.65
$\Delta\pi_{t-3}^*$	0.20	3.27	---	---	0.21	1.61	0.44	2.08
$\Delta\pi_{t-4}^*$	-0.01	-0.12	---	---	-0.03	-0.22	-1.93	-8.90
$\Delta\pi_{t-5}^*$	0.15	2.33	---	---	-0.10	-0.82	1.61	6.84
$\Delta\pi_{t-6}^*$	-0.03	-0.49	---	---	-0.10	-0.78	1.16	5.12
$\Delta\pi_{t-7}^*$	0.14	2.13	---	---	0.12	0.99	-0.35	-1.48
$\Delta\pi_{t-8}^*$	0.08	1.30	---	---	0.12	0.97	-1.52	-6.75
$\Delta\pi_{t-9}^*$	0.02	0.39	---	---	---	---	0.70	3.44
$\Delta\pi_{t-10}^*$	0.08	1.29	---	---	---	---	-0.16	-0.85
$\Delta\pi_{t-11}^*$	0.07	1.17	---	---	---	---	0.65	4.42
$\Delta\pi_{t-12}^*$	-0.35	-6.00	---	---	---	---	-0.50	-5.50
$\Delta\pi_{t-13}^*$	---	---	---	---	---	---	0.10	3.12
$(\pi - \pi^*)_{t-1}$	0.02	1.89	0.15	1.68	0.06	1.24	0.00	0.59

Table 6: Granger Causality Tests for Headline and Denoised Inflation

Null Hypothesis:	F-Statistic	Prob.
$\pi_t$ does not Granger Cause $\pi_t^*$	1.57	0.09
$\pi_t^*$ does not Granger Cause $\pi_t$	44.14	0.00

**Table 7: Turning Points and Cross Points**

	July 1992-June 2017				July 2008-June 2017			
	$\pi_t^{*N}$	$\pi_t^{*T}$	$\pi_t^{*R}$	$\pi_t^{*D}$	$\pi_t^{*N}$	$\pi_t^{*T}$	$\pi_t^{*R}$	$\pi_t^{*D}$
No's of Turning Points	97	---	---	18	38	42	38	5
No's of cross point ( $\pi_t$ vs $\pi_t^*$ )	42	---	---	87	13	20	14	30

**Table 8: Bi-variate Granger Non-Causality Analysis between Monetary Growth and Core Inflation**

Core Measure	Null Hypothesis	P-value*
NFNE	Core inflation does not cause M2 growth	0.07
	M2 growth does not cause core inflation	0.17
Trimmed	Core inflation does not cause M2 growth	0.27
	M2 growth does not cause core inflation	0.12
RSC	Core inflation does not cause M2 growth	0.77
	M2 growth does not cause core inflation	0.45
Denoised	Core inflation does not cause M2 growth	0.81
	M2 growth does not cause core inflation	<b>0.04</b>

\* P-value less than 0.05 means the null hypothesis is rejected. Lags are selected on using AIC.

**Table 9: Spearman's Correlation Coefficient between Monetary Growth and Core Inflation**

Core Measure	Lag	Correlation Coefficient
NFNE	Contemporaneous	0.12*
	12 months	0.50*
	24 months	0.41*
Trimmed	Contemporaneous	0.29*
	12 months	0.35*
	24 months	-0.05
RSC	Contemporaneous	0.15*
	12 months	0.32*
	24 months	-0.05
Denoised	Contemporaneous	<b>0.19*</b>
	12 months	<b>0.42*</b>
	24 months	<b>0.33*</b>

\* Significantly different from zero.

**Table 10: Correlation Coefficient between Monetary Growth and Excluded Inflation**

Excluded Part	Lag	Correlation Coefficient
Food and Energy Inflation	Contemporaneous	0.21*
	12 months	0.32*
	24 months	0.21*
20-percent excluded items' Inflation	Contemporaneous	0.14*
	12 months	0.16*
	24 months	-0.23*
Relatively Unstable Components' Inflation	Contemporaneous	0.29*
	12 months	0.23*
	24 months	-0.22*
Inflation Noise	Contemporaneous	-0.03
	12 months	0.07
	24 months	-0.07

\* Significantly different from zero.

**Table 11: Granger Non-Causality Analysis between 'Food & Energy' and Expected Inflation<sup>§</sup>**

Null Hypothesis	Lags Included	P-value*
Food & Energy inflation does not Granger Cause Expected inflation	14	0.00
Expected inflation does not Granger Cause Food & Energy inflation		0.21

\* P less than 0.05 mean the null hypothesis is rejected. Lags are selected on the basis of AIC.

<sup>§</sup> Period=Jan 2013 to Jan 2018. Bia-monthly inflation expectation data is taken from SBP.